



# Workshop on Aperiodic Order: Enumeration Problems, Dynamics, and Topology

24–27 May 2017

Faculty of Mathematics Bielefeld University Lecture Room: V2-105/115

This workshop is part of the DFG-funded CRC 701 Spectral Structures and Topological Methods in Mathematics at Bielefeld University

Organisers: Michael Baake, Franz Gähler and Christian Huck www.math.uni-bielefeld.de/baake/apo/index.html

### Schedule: Wednesday May 24th

- 10:00-10:15 **opening**
- 10:15–11:00 **Nicolae Strungaru** (MacEwan University, Edmonton) On the Fourier transformability of a strongly almost periodic measure
- 11:00-11:30 Coffee Break (Room V3-201)
- 11:30–12:30 **Dan Rust** (Universität Bielefeld) Topological entropy of random substitution subshifts
- 12:30-14:00 Lunch Break
- 14:00-15:00 Individual Discussion
- 15:00–16:00 **Timo Spindeler** (Universität Bielefeld) Diffraction of the random Fibonacci substitution
- 16:00–16:30 Coffee Break (Room V3-201)
- 17:00–18:00 **David Damanik (Kolloquium)** (Rice University, Houston) The 1D Anderson Model: From Fürstenberg's Theorem to Spectral and Dynamical Localization via Large Deviation Estimates

Workshop on Aperiodic Order:

Enumeration Problems, Dynamics, and Topology

### Schedule: Thursday May 25th

- 10:00–11:00 Christoph Richard (Universität Erlangen-Nürnberg) Periods and factors of weak model sets
- 11:00-11:30 Coffee Break (Room V3-201)
- 11:30–12:30 **Gerhard Keller** (Universität Erlangen-Nürnberg) *B*-free dynamics: A view through the window
- 12:30-14:00 Lunch Break at Restaurant Rosenhof
- 14:00-15:00 Individual Discussion
- 15:00–16:00 **Pierre Arnoux** (Institut de Mathématique de Luminy) S-adic systems associated with strongly convergent continued fractions
- 16:00-16:30 Coffee Break (Room V3-201)
- 16:30–17:30 **Robbert Fokkink** (TU Delft) Towards a topological Takens theorem

### Schedule: Friday May 26th

- 10:00–11:00 Antoine Julien (Nord University, Bodø) Homeomorphisms between tiling spaces
- 11:00-11:30 Coffee Break (Room V3-201)
- 11:30–12:00 Scott Balchin (University of Leicester) Computations for 1-Dimensional Tiling Spaces
- 12:00–12:30 Marc-Christian Zimmermann (TU Dortmund) An enumeration problem for finite quadratic forms
- 12:30-14:00 Lunch Break
- 14:00-15:00 Individual Discussion
- 15:00–16:00 Alex Clark (University of Leicester) Aperiodic order and induced structures
- 16:00–16:30 Coffee Break (Room V3-201)
- 16:30-17:30 **Mike Whittaker** (University of Glasgow)  $C^*$ -algebras associated with aperiodic tilings and their classification

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### Schedule: Saturday May 27th

- 10:00–11:00 Aurelia Bartnicka (Nicolaus Copernicus University, Toruń) Minimal and proximal *B*-free systems
- 11:00-11:30 Coffee Break (Room V3-201)
- 11:30–12:30 Mariusz Lemańczyk (Nicolaus Copernicus University, Toruń) Möbius disjointness for models of an ergodic sytems and beyond
- 12:30-13:30 Lunch Break
- 13:30–14:00 **Daniel Sell** (University of Jena) About the Boshernitzan condition for simple Toeplitz subshifts
- 14:00–15:00 **Peter Zeiner** (Bielefeld University) Enumeration of lattices and embedded Z-modules

### Abstracts

### Pierre Arnoux (Institut de Mathématique de Luminy)

### S-adic systems associated with strongly convergent continued fractions

It is well known that the usual continued fraction is related to the dynamics of circle rotations, and to the geodesic flow on the modular surface. We show that a similar construction can be carried out for almost everywhere strongly convergent continued fractions, connecting them with the dynamics of a family of torus translations, and a flow on an homogeneous space. The main ingredient is a family of S-adic symbolic systems linked to the continued fraction, which can be shown to have good symbolic models.

### Scott Balchin (University of Leicester)

### Computations for 1-Dimensional Tiling Spaces

In this talk I will cover some of the algorithms used in Grout to compute invariants for 1dimensional tiling spaces. In particular, I will describe the ingredients for the implementation of a finite check for recognisability. Moreover, I will discuss some changes and additions in upcoming versions of Grout. This is joint work with Dan Rust.

### Aurelia Bartnicka (Nicolaus Copernicus University)

### Minimal and proximal *B*-free systems

Let  $\mathscr{B} \subset \mathbb{N} \setminus \{1\}$  and  $\mathcal{M}_{\mathscr{B}} = \bigcup_{b \in \mathscr{B}} b\mathbb{Z}$  denote the set of multiples. We will consider the subshift given by the closure of the orbit of the 0–1-sequence  $\eta$  whose support is the set of  $\mathscr{B}$ -free numbers  $\mathcal{F}_{\mathscr{B}} = \mathbb{Z} \setminus \mathcal{M}_{\mathscr{B}}$ . We will be interested in the case when the  $\mathscr{B}$ -free subshift is minimal or proximal. We will show that in the first case the corresponding  $\mathscr{B}$ -free subshift is Toeplitz, in that  $\eta$  itself is Toeplitz. We will also describe a periodic structure. We will formulate some open questions.

### Alex Clark (Nicolaus Copernicus University)

### Aperiodic Order and Induced Structures

We will consider how to use the global structure of tiling spaces to construct new classes of spaces that have interesting features. In particular, we will discuss how to produce examples of hereditarily indecomposable continua that occur as minimal sets with positive entropy of diffeomorphisms. Previously, such spaces were only known to support homeomorphisms with 0 or infinite entropy. This technique could be used more generally to transfer the recurrence properties of aperiodic tilings to more general classes of spaces. (Based on joint work with J. Boronski and P. Oprocha)

### David Damanik (Rice University)

# The 1D Anderson Model: From Fürstenberg's Theorem to Spectral and Dynamical Localization via Large Deviation Estimates

Anderson's Nobel Prize-winning work showed that a random environment can localize a quantum state for all times. In general dimensions, and without assuming sufficient regularity of the single-site distribution, this can be rigorously shown only via an intricate multi-scale analysis. In one dimension, the positivity of the Lyapunov exponent that can be shown using Fürstenberg's Theorem has for a long time been expected to imply Anderson localization rather directly. However, for singular distributions such as the Bernoulli case, no direct proof was available and

a multi-scale analysis was still necessary. This talk will explain the history of this problem and its proof, the obstacles in finding a direct path from Fürstenberg to Anderson, and recent work that does in fact find the desired direct path and opens the door for new results that were previously inaccessible in situations where no multi-scale analysis is available.

#### Robbert Fokkink (TU Delft)

#### Towards a topological Takens theorem

Floris Takens proved that smooth dynamical systems embed generically in  $\mathbb{R}^n$  by a delay embedding. More precisely, if h is a diffeomorphism on a manifold M and f is differentiable, then the delay map  $T(x) = (f(x), f(hx), \dots, f(h^m(x)))$  is an embedding, if  $m > 2\dim(M)$ , generically in f and h. Takens' theorem initiated the geometric study of time-series. Extending the theorem from manifolds to arbitrary topological spaces X seems to be difficult. It is possible to show that Takens' theorem holds in the very special case that X is the pseudo-arc. This is joint work with Jan Boronski.

#### Antoine Julien (Nord University)

#### Homeomorphisms between tiling spaces

It is a natural question to ask: "when two tiling spaces are equivalent, what can be said of the underlying tilings?" When considering homeomorphisms between spaces of tilings of finite local complexity, we can give a few answers: notably, their complexity and repetitivity functions are equivalent up to rescaling. The underlying reason is that a homeomorphism between tiling spaces is essentially a "time-change" of the tiling dynamical system, composed with a conjugacy. In combinatorial terms, it is induced by a shape-deformation of the tiles, and can be parametrized by a cohomology element. When the homeomorphism in addition respects the longitudinal differentiable structure, we can strengthen our results.

Part of this work is in collaboration with Lorenzo Sadun.

### Gerhard Keller (Universität Erlangen-Nürnberg)

#### B-free dynamics: A view through the window

Let B be an infinite set of positive integers. Exploiting the fact that B-free sets are special cases of weak model sets, we relate topological and measure theoretic properties of the "window" to arithmetic properties of the set B. Most of this is from joint work with S. Kasjan and M. Lemańczyk, motivated in parts by joint work with C. Richard.

### Mariusz Lemańczyk (Nicolaus Copernicus University)

#### Möbius disjointness for models of an ergodic sytems and beyond

I plan to talk about the most recent results on Sarnak's conjecture on Möbius disjointness, especially focusing on an answer to the question whether if a model of an ergodic automorphism is Möbius disjoint then all of them are. The talk is based on a joint work with H. el Abdalauoi, J. Kulaga-Przymus and T. de la Rue.

### Christoph Richard (Universität Erlangen-Nürnberg)

### Periods and factors of weak model sets

Model sets can be described dynamically using their hull. In particular, for prominent examples, the hull is an almost one-to-one extension of the torus underlying the model set, and the maximal equicontinuous factor of the hull is the torus. We investigate dynamical properties of certain weak model sets both from a topological and from a measure-theoretic perspective. This substantially generalises well-known results about model sets to weak model sets having a window which is "not far from being compact". (Joint work with Gerhard Keller, arXiv:1702.02383)

### Dan Rust (Universität Bielefeld)

### Topological entropy of random substitution subshifts

Random substitutions are a recent generalisation of the classical notion of a substitution, in which a letter of the alphabet is mapped to a finite set of possible words rather than a single determined word. We are now beginning to understand the dynamical properties of subshifts associated to these random substitutions. I will give an overview of some recent developments stemming from joint work with Timo Spindeler, focussing on the topological entropy of the subshifts.

### Daniel Sell (Friedrich Schiller University)

### About the Boshernitzan condition for simple Toeplitz subshifts

One-dimensional Toeplitz words are constructed from periodic words with undetermined positions by successively filling these "holes" with other periodic words. In this talk, we will discuss how some combinatorial properties of so called simple Toeplitz words are related to the Boshernitzan condition of the associated subshift. This condition implies Cantor spectrum of Lebesgue measure zero for Jacobi operators on the subshift.

### Timo Spindeler (Universität Bielefeld)

### Diffraction of the random Fibonacci substitution

It is well-known that the symbolic hull of a primitive substitution (together with the  $\mathbb{Z}$ -action) has zero entropy, and that the corresponding diffraction measure is pure point. In this talk, we are going to study an example of a primitive random substitution, which gives rise to a symbolic hull with positive entropy and a diffraction spectrum of mixed type.

### Nicolae Strungaru (MacEwan University)

### On the Fourier transformability of a strongly almost periodic measure

In this talk we provide a necessary and sufficient condition for a strongly almost periodic measure to be Fourier transformable. Given a tempered measure on the Euclidian space, we also discuss the connection between its Fourier transformability as a measure and as a tempered distribution.

### Mike Whittaker (University of Glasgow)

#### $C^{\ast}\mbox{-algebras}$ associated with a periodic tilings and their classification

I will introduce Kellendonk's  $C^*$ -algebra associated with an aperiodic tiling and show that these algebras are classifiable by the Elliott invariant (essentially K-theory). Classification follows from showing that Kellendonk's tiling groupoids have finite dynamic asymptotic dimension and the proof is mainly topological. This talk will not assume a background in  $C^*$ -algebras and I will introduce all the relevant notions. This is joint work with Luke Hamblin.

#### Peter Zeiner (Universität Bielefeld)

#### Enumeration of lattices and embedded $\mathbb{Z}$ -modules

We want to count and classify certain sublattices of a given lattice, and more generally, submodules of  $\mathbb{Z}$ -modules embedded in  $\mathbb{R}^d$ . Emphasis is put on problems motivated from crystallography, which includes similar sublattices and coincidence site lattices. We use methods from algebra and number theory to count them. In addition, we determine the asymptotic behaviour of the corresponding counting functions by means of suitable Dirichlet series

### Marc-Christian Zimmermann (TU Dortmund)

#### An enumeration problem for finite quadratic forms

Regular quadratic modules over residue class rings of the integers arise naturally as quotients L/cL of an integral lattice L. In this situation certain interesting sublattices, e.g. similar sublattices, of L correspond to maximal totally isotropic submodules of the quotient. Motivated by this, a cancellation law for such submodules of regular quadratic modules over residue class rings of the integers is derived. Applications of this cancellation law include a complete classification, and a method to count and enumerate maximal totally isotropic submodules.

# Participants

Pierre Arnoux	(Institut de Mathématique de Luminy, France)
Michael Baake	(Bielefeld University, Germany)
Scott Balchin	(University of Leicester, United Kingdom)
Aurelia Bartnicka	(Nicolaus Copernicus University, Toruń, Poland)
Alex Clark	(University of Leicester, United Kingdom)
David Damanik	(Rice University, Houston, USA)
Fabian Dreher	(University of Bremen, Germany)
Robbert Fokkink	(TU Delft, Netherlands)
Franz Gähler	(Bielefeld University, Germany)
Philipp Gohlke	(Bielefeld University, Germany)
Christian Huck	(Bielefeld University, Germany)
Tobias Jakobi	(Bielefeld University, Germany)
Antoine Julien	(Nord University, Norway)
Gerhard Keller	(Friedrich-Alexander University Erlangen-Nürnberg, Germany)
Holger Kösters	(Bielefeld University, Germany)
Marius Lemańczyk	(Nicolaus Copernicus University, Toruń, Poland)
Chrizaldy Neil Mañibo	(Bielefeld University, Germany)
Arne Mosbach	(University of Bremen, Germany)
Christoph Richard	(Friedrich-Alexander University Erlangen-Nürnberg, Germany)
Timo Rosnau	(Bielefeld University Germany)
Dan Rust	(Bielefeld University, Germany)
Rudolf Scharlau	(TU Dortmund, Germany)
Daniel Sell	(Friedrich Schiller University Jena, Germany)
Timo Spindeler	(Bielefeld University, Germany)
Malte Steffens	(University of Bremen, Germany)
Nicolae Strungaru	(MacEwan University, Edmonton, Canada)
Michael Whittaker	(University of Glasgow, United Kingdom)
Peter Zeiner	(Bielefeld University, Germany)
Marc-Christian Zimmermann	(TU Dortmund, Germany)

## Tram map



# Campus map

