Integrable quantum lattice spin models in two and three dimensions

Integrable lattice models are important tools in many branches of physics: they open the possibility to describe analytically simultaneously regimes of weak and strong coupling (and of order and disorder and singularities at criticality), avoiding the limitations of perturbative approaches. The starting concept for constructing integrable 2-dimensional lattice spin models has been mainly the quadratic Yang-Baxter-type algebra of the monodromy matrices.

Focusing first on 2-dimensional models, we shortly review the standard quantum inverse scattering method of the Leningrad school. Mathematically rather interesting problems are encountered in solving the Integrabel Chiral Potts model. Mostly, the physics has to be inferred from the spectrum only, as results on state vectors and correlation functions are limited and complicated. We shall also mention the very recent Canberra Master solution.

Then we consider the much more involved 3-dimensional case based on the tetrahedron equation. Here we sketch how the algebraic integrability conditions follow from geometrical consistency properties, a point of view attracting presently much attention. Up to now, only three classes of integrable 3-dimensional lattice models have been found. In the cyclic solution class the Boltzmann weights are cross ratios of quantum dilogarithms at root of unity.