

The simplest kind of discrete time lattice system is a quantum walk. The Hilbert space of such a system is just the space of square summable sequences on the lattice, tensored with some finite-dimensional “internal” degree of freedom. The single-step unitary operator allows only finite jumps. We begin with the translation invariant case, in which diagonalization leads to dispersion curves winding around a torus. The spectrum is then purely absolutely continuous and the limiting distribution of $X(t)/t$ (position in ballistic scaling) converges to the distribution of group velocity. We describe the method in detail, which also yields an exponential tail estimate for the total probability outside the set of allowed velocities. We then further extend the method to cover randomness in time, i.e., a global change of the unitary in each step, driven by some external Markov process. In this case the spreading becomes diffusive, i.e. $X \sim \sqrt{t}$, and the diffusion constants follow from a second order perturbation analysis. In contrast, when the operation is randomized over lattice sites (and quenched) one gets Anderson localization. With randomness in both time and space one gets back to diffusive scaling. Hence randomness in time slows down a ballistic evolution, but speeds up a localized one.

We then discuss molecule formation, when two walking particles in one dimension interact via an on-site interaction phase. The molecular states are defined by the property that the two particles stay close together with bounds valid for all times. Their center of mass then performs a quantum walk in its own right. Typically the compounds are slower than the free constituents, but the opposite is also possible.

The classical notion of recurrence, i.e., return to the starting point with probability one has different quantum generalizations, depending on the way the return of the particle is monitored. If this is done by a projective measurement, one finds recurrence iff the spectrum has no absolutely continuous part. The expected return time in that case is infinite or an integer. Actually, this integer is the dimension of the cyclic subspace generated by the initial state, but also has an interpretation as a winding number. Time permitting, an alternative approach for the absolutely continuous spectrum, namely by covariant arrival time measurements will also be discussed.