Peter Zeiner

Faculty of Mathematics,
Bielefeld University

Bielefeld, April 5, 2007

Optimization and Dynamics

Summer term 2007

Exercises – sheet 1

(1) Consider the difference equation

$$x_{n+1} = -x_n \tag{1}$$

with initial value $x_0 \in \mathbb{R}$. Find an explicit expression for the solution. What are the fixed points? Are there periodic orbits?

(2) Consider the difference equation

$$x_{n+1} = ax_n(1 - x_n), \tag{2}$$

where $0 < a < 1$.

(a) Show that $\{x_n\}_{n=0}^{\infty}$ is a strictly monotone decreasing sequence for $0 < x_0 < 1$.

(b) What happens for $x_0 = 0$ and $x_0 = 1$?

(3) Consider the difference equation

$$x_{n+1} = 4x_n(1 - x_n). \tag{3}$$

Let $x_0 = (\sin \phi_0)^2$. Calculate the period of the orbits $\{x_0, x_1, x_2, x_3, \ldots\}$ for $\phi_0 = \frac{\pi}{7}$, $\frac{3\pi}{7}$, and $\frac{\pi}{9}$.

(4) Consider again the difference equation

$$x_{n+1} = ax_n(1 - x_n), \tag{4}$$

where we assume $a \geq 1$ now.

(a) Show that $x_n < 0$ for all $n \in \mathbb{N}$ if $x_0 < 0$.

(b) Show that $\{x_n\}_{n=0}^{\infty}$ is a strictly monotone decreasing sequence. Does it converge?

(c) What happens if $x_0 > 1$?

(5) Solve the differential equation

$$\dot{x}(t) = x^2(t) \tag{5}$$

for the initial value $x(0) = x_0 > 0$ in the time interval $0 \leq t < \frac{1}{x_0}$. Sketch the graph! What happens at $t = \frac{1}{x_0}$? Does the same problem arise if x_0 is negative?