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Optimization and Dynamics

Summer term 2007

Assignment sheet 10

(32) Consider the differential equation

$$x' = 2|x|^{1/2}$$

with initial condition $x(0) = -1$.

(a) Prove that there are infinitely many solutions, i.e. show that

$$x_\alpha(t) := \begin{cases} -(1-t)^2 & \text{for } t \leq 1 \\ 0 & \text{for } 1 \leq t \leq \alpha \\ (t-\alpha)^2 & \text{for } \alpha \leq t \end{cases}$$

is a solution for every $\alpha > 1$.

(b) Are the functions $x_\alpha(t)$ differentiable, C^1–functions, C^2–functions, C^∞–functions?

(c) Does it make sense to define a function $x_\infty(t)$? If no, why not? If yes, how has it to be defined?

(d) In the lecture we have discussed the equation

$$x' = x^{2/3}$$

with initial conditions $x(0) = 0$ and we have constructed a finite number of different solutions. Do there exist further solutions, maybe even an infinite number? If yes, how do they look like? If no, why not?

(33) Consider the differential equation

$$x'(t) = t\,x(t)$$

with initial condition $x(0) = x_0$.

(a) Solve the differential equation. Is there a solution for all $x_0 \in \mathbb{R}$?

(b) Is the solution unique?

Turn around please!

(34) Consider the differential equation

$$x'(t) = t\,x(t)^2$$

with initial condition $x(0) = x_0$.

(a) Solve the differential equation. Is there a solution for all $x_0 \in \mathbb{R}$? Is it defined on $\mathbb{R}$ or just on a subset of $\mathbb{R}$?

(b) Is the solution unique?

Please hand in until 15.6.2007.