Faculty of Mathematics, Bielefeld University

Optimization and Dynamics

Summer term 2007

Assignment sheet 11

(35) Consider the initial value problem

$$x' = -\frac{1}{2x},$$
 $x(t_0) = x_0$

- (a) Where is the function $f(x) = -\frac{1}{2x}$ defined? Does it satisfy a local Lipschitz condition (on which intervals)?
- (b) Solve the given initial value problem. What are the maximal intervals $I(x_0, t_0)$ on which the solution is defined? What is the general solution $\Phi(t, x_0, t_0)$? Where is it defined?
- (c) Prove that $\Phi(t, x_0, t_0)$ depends only on x_0 and $t t_0$, i.e. show that $\Phi(t, x_0, t_0) = \varphi(t t_0, x_0)$ for a suitable function φ .
- (d) Show that $x(t) = \varphi(t, x_0)$ is the solution of the initial value problem

$$x' = -\frac{1}{2x}, \qquad \qquad x(0) = x_0$$

- (e) Prove $\varphi(s+t,x) = \varphi(s,\varphi(t,x))$.
- (36) Do the same for

$$x' = 2x^3, \qquad \qquad x(t_0) = x_0$$

(37) Consider the initial value problem

$$x'(t) = t x(t),$$
 $x(t_0) = x_0$

- (a) Where is the function f(x,t) = tx defined? Does it satisfy a local Lipschitz condition (on which intervals)?
- (b) Solve the given initial value problem. What are the maximal intervals $I(x_0, t_0)$ on which the solution is defined? What is the general solution $\Phi(t, x_0, t_0)$? Where is it defined? Can it be written as $\Phi(t, x_0, t_0) = \varphi(t t_0, x_0)$?
- (c) Prove $\Phi(t, x_0, t_0) = \Phi(t, \Phi(t_1, x_0, t_0), t_1)$ for $t \ge t_1 \ge t_0$.

Turn around please!

(38) Consider the two dimensional discrete dynamical system $x_{n+1} = Ax$, where

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}.$$

Determine the Lyapunov exponent. Show that it is independent of the initial value x_0 .

Please hand in until 22.6.2007.