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Optimization and Dynamics

Summer term 2007

Assignment sheet 11

(35) Consider the initial value problem

$$x' = -\frac{1}{2x}, \qquad x(t_0) = x_0.$$

(a) Where is the function $f(x) = -\frac{1}{2x}$ defined? Does it satisfy a local Lipschitz condition (on which intervals)?

(b) Solve the given initial value problem. What are the maximal intervals $I(x_0, t_0)$ on which the solution is defined? What is the general solution $\Phi(t, x_0, t_0)$? Where is it defined?

(c) Prove that $\Phi(t, x_0, t_0)$ depends only on x_0 and $t - t_0$, i.e. show that $\Phi(t, x_0, t_0) = \varphi(t - t_0, x_0)$ for a suitable function φ.

(d) Show that $x(t) = \varphi(t, x_0)$ is the solution of the initial value problem

$$x' = -\frac{1}{2x}, \qquad x(0) = x_0.$$

(e) Prove $\varphi(s + t, x) = \varphi(s, \varphi(t, x))$.

(36) Do the same for

$$x' = 2x^3, \qquad x(t_0) = x_0.$$

(37) Consider the initial value problem

$$x'(t) = t\,x(t), \qquad x(t_0) = x_0.$$

(a) Where is the function $f(x, t) = tx$ defined? Does it satisfy a local Lipschitz condition (on which intervals)?

(b) Solve the given initial value problem. What are the maximal intervals $I(x_0, t_0)$ on which the solution is defined? What is the general solution $\Phi(t, x_0, t_0)$? Where is it defined? Can it be written as $\Phi(t, x_0, t_0) = \varphi(t - t_0, x_0)$?

(c) Prove $\Phi(t, x_0, t_0) = \Phi(t, \Phi(t_1, x_0, t_0), t_1)$ for $t \geq t_1 \geq t_0$.

Turn around please!

(38) Consider the twodimensional discrete dynamical system $x_{n+1} = Ax$, where

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}.$$

Determine the Lyapunov exponent. Show that it is independent of the initial value x_0.

Please hand in until 22.6.2007.