Peter Zeiner

Faculty of Mathematics,
Bielefeld University

Bielefeld, April 11, 2007

Optimization and Dynamics

Summer term 2007

Assignment sheet 2

(6) Consider the difference equation

$$x_{n+1} = ax_n(1 - x_n), \tag{1}$$

where $a = 2$.

(a) Discuss its solutions (fixed points, periodic orbits, increasing (decreasing) solutions, stability, ...)

(b) Compare with the case $1 < a < 2$.

(7) Discuss the dynamical systems determined by the family of functions

$$f_c(x) = x^2 + c. \tag{2}$$

Hint: Discuss the three cases $c < \frac{1}{4}$, $c = \frac{1}{4}$, and $c > \frac{1}{4}$ seperately.

(8) Let $f : X \to X$ be a continuous function and let $O^+ = \{x_0, x_1, \ldots, x_{p-1}, x_0, \ldots\}$ be a periodic orbit of period p. Show that O^+ is an attractor if and only if x_0 is an attracting fixed point of the dynamical system generated by f^p.

(9) Consider the dynamical system determined by

$$f(x) = \begin{cases} 1 & \text{for } \frac{1}{4} < x < 1 \\ \frac{1}{2} & \text{for } x = \frac{1}{4}, 1 \\ 0 & \text{otherwise .} \end{cases} \tag{3}$$

(a) Sketch the graph and the phase portrait.

(b) Find the fixed points and discuss their stability. Are they attractors or repellers?

(c) Find the periodic orbits and discuss their stability. Are they attractors or repellers?

(d) Discuss the stability of the fixed points of f^2? Compare with (c) and compare with lemma 1.13.