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Optimization and Dynamics

Summer term 2007

Assignment sheet 5

(18) Discuss the bifurctions at $a = -1$ for the following dynamical system

$$f_a(x) = x + \sin x + a.$$

What are the fixed points for $a < -1$ and $a > -1$? Discuss their stability, and sketch the bifurcation diagram. What kind of bifurcation is it?

For which value of a do you expect another bifurcation?

(19) Discuss the bifurctions at $a = 1$ for the following dynamical system

$$f_a(x) = a \sin x.$$

What are the fixed points for $a < 1$ and $a > 1$? Discuss their stability, and sketch the bifurcation diagram. What kind of bifurcation is it?

For which value of a do you expect another bifurcation?

(20) Proof theorem 3.7.

Hint: Proceed as in the proof of theorem 3.5. Define

$$H(a,x) = \begin{cases} \dfrac{F(a,x) - x}{x - \bar{x}} & \text{if } x \neq \bar{x} \\ \lim\limits_{y \to \bar{x}} \dfrac{F(a,y) - y}{y - \bar{x}} & \text{if } x = \bar{x} \end{cases}$$

and apply the implicit function theorem to $H(a,x)$. In order to calculate $\frac{\partial^n H}{\partial x^n}(a, \bar{x})$ expand $F(a,x)$ in a Taylor series about $\bar{x}$.

Please hand in until 10.5.2007.