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Bielefeld, May 11, 2007

# Optimization and Dynamics

## Summer term 2007

## Assignment sheet 6

**(21)** Prove that
$$F(a,x) = ax(1-x)$$
has a period doubling bifurcation at $a = 3$.

**(22)** Consider the logistic map
$$F(a,x) = ax(1-x)$$
again.

(a) What happens at $a = 0$? How many fixed points are there? What is their stability, are they attracting or repelling? Are there any eventually fixed or periodic points?

(b) Find a bifurcation for $a < 0$. What kind of bifurcation is it?

(c) Do you expect further bifurcations? What do you guess? How could the bifurcation diagram look like?

**(23)** Consider the family of maps
$$f_a(x) = a + e^{-x} - 1.$$

(a) Proof that there is exactly one fixed point for any $a$. *Hint: Use the intermediate value theorem. What do you know about the derivative of $f_a$?*

(b) Is there a bifurcation? If no, why not. If yes, what kind of bifurcation is it?

**(24)** Consider the dynamical system given by the function
$$f(x) = \begin{cases} 1 & \text{for } x = 0 \\ 2 & \text{for } x = 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Are there any (eventually) fixed points? Are there any (eventually) periodic orbits? If yes, what is their period?

(b) What does this example show?

*Please hand in until 18.5.2007.*