Faculty of Mathematics, Bielefeld University

Optimization and Dynamics

Summer term 2007

Assignment sheet 9

(31) Consider the dynamical system given by the function

$$F(x,y) = \begin{pmatrix} \frac{1}{2}(x+x^3) \\ 2y\frac{1}{1+2x^2} \end{pmatrix}.$$

- (a) Determine the fixed points.*Hint: Consider the equation for x first.*
- (b) Show that all fixed points are saddle points.
- (c) Sketch the phase portrait of F.
 Hint: The stable and unstable manifolds are all parallel to the axes.
- (d) Choose one of the saddle points. Prove that its stable and unstable manifolds are parallel to the axes, i.e. show that there are open intervals W_s and W_u parallel to the axes such that $F(W_{u,s}) \subseteq W_{u,s}$ and $\lim_{n \to \infty} F^n(x)$ for all $x \in W_s$ and $\lim_{n \to \infty} F^{-n}(x)$ for all $x \in W_u$.