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Optimization and Dynamics

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Assignment sheet 9

(31) Consider the dynamical system given by the function

$$F(x,y) = \begin{pmatrix} \frac{1}{2}(x+x^3) \\ 2y\frac{1}{1+2x^2} \end{pmatrix}.$$

(a) Determine the fixed points.
Hint: Consider the equation for x first.

(b) Show that all fixed points are saddle points.

(c) Sketch the phase portrait of F.
Hint: The stable and unstable manifolds are all parallel to the axes.

(d) Choose one of the saddle points. Prove that its stable and unstable manifolds are parallel to the axes, i.e. show that there are open intervals W_s and W_u parallel to the axes such that $F(W_{u,s}) \subseteq W_{u,s}$ and $\lim\limits_{n\to\infty} F^n(x)$ for all $x \in W_s$ and $\lim\limits_{n\to\infty} F^{-n}(x)$ for all $x \in W_u$.

Please hand in until 8.6.2007.