ON RATIONAL POINTS OF LINEAR ALGEBRAIC GROUPS

Let $k$ be a field with characteristic $0$. The $k$-equivalence on the group $G(k)$ of $k$-rational points of $G$ is an obstruction to the parametrization of $G(k)$; in the case of tori, it is the only one. We recall the following conjecture: Is $G(k)/k$ abelian? For all known cases, $G(k)/k$ is abelian (e.g., $SL(2)/k$, $Spin(9,1)$).

We discuss the following examples.

- $K_{	ext{norm}}$, i.e., equation $N_{k'/k}(x) = 1$ for any field extension $k'/k$.
- $SL(2)$ after Bass.
- $PSO(q)$ (after Platonov, talk of the 10th December 1993).

About the last example, we have the following additional result.

Theorem: There exists a field $k$, and a quadratic form $q(x)$ of rank 1 with discriminant $1$ such that $PSO(q)(k)/k \neq 1$.

We discuss although the Manin’s finiteness conjecture.

Conjecture: Let $G/k$ be a linear algebraic group defined over a field $k$ finitely generated over $Q$. Then $G(k)/k$ is finite.

In the case of tori, this conjecture was proven by Colliot-Thélène-Szamuely. And in the case of linear groups over number fields, we have the:

Theorem. Let $G/k$ be a linear algebraic group defined over a number field $k$. Then $G(k)/k$ is finite.
Exceptional sets in analysis and probability theory

C. B. Dykinson

January 15, 1969

It is well-known that a subset of $\mathbb{R}^d$ is not hit, or
by the Brownian motion if and only if its classical
capacity is equal to 0. More recently, it was established
that the sets not hit by the super-Brownian motion
can be characterized similarly in terms of suitable Besel
capacities. These sets are also removable singularities for solutions of

certain semilinear PDEs. The most recently, analogous con-
nections were proved for boundary singularities. The theory is
rather complete for domains with a smooth boundaries.

There are still challenging open problems for general domains
for smooth manifolds) and the corresponding Martin boundaries.
Der Chem-Connes Charakter in der nichtkommutativen Geometrie.

J. Cuntz, 15.1.99

Für geeignete Kategorien $\mathcal{C}$ von Algebren über $\mathbb{C}$ (oder $\mathbb{R}$) können bivariante $K$-Theorie-Gruppen $kk_{\mathcal{C}}(A,B)$ und bivariante (periodische) zyklische Homologiegruppen $H_{\mathcal{C}}^i (A,B)$, $A, B \in \mathcal{C}$ und $i \in \mathbb{Z}/2$ definiert werden. Diese Theorien verallgemeinern die klassische $K$-Theorie und die Rham Theorie und liefern gleichzeitig einen ganz neuen Zugang.

Wir beschreiben einen bivarianten multiplikativen "Chem-Connes-Charakter" $Ch: kk_{\mathcal{C}}(A,B) \to H_{\mathcal{C}}^i (A,B)$ und diskutieren seine Eigenschaften.

J. Cuntz
Universität Münster
Homologische Theorie
in der Kohomologie theorie s-orthogonaler Gruppen

Hans-Werner Heun 29.1.99

Der mod p Kohomologie einer (s-) orthogonalen Gruppe ist nur in sehr wenigen Fällen explizit bekannt, z. B. für $\text{SL}(n, \mathbb{Z})$, das für $n \leq 3$. Wir beschreiben Fortschritte im Verständnis der Kohomologie s-orthogonalen Gruppen (z. B. $\text{SL}(n, \mathbb{Z}/p\mathbb{Z})$, $\text{O}_n(\mathbb{Z}/p\mathbb{Z})$), die mit modernen Methoden aus der Homologietheorie klassifizierende Räume erhalten werden sind.

Hans-Werner Heun

Strasbourg
Gauss periods in finite fields

Joachim von zur Gathen
Paderborn

Carl Friedrich Gauss invented his periods for his construction of the regular 17-gon. Taken modulo prime numbers (or prime ideals), these Gauss periods sometimes provide normal elements in finite field extensions. We present two applications. The first one is to find elements in finite fields of exponentially large order with an efficient algorithm; this is a step towards the open problem of finding primitive elements efficiently. The second application is fast exponentiation. For fields like \( \mathbb{F}_{2^n} \), this yields algorithms that are fast both in theory and in practice. Finally, we present a more general construction than Gauss' original one, which gives a wider range of applicability.

5 February 1999

[Signature]
Polynomial Structures on Polycyclic Groups.

Karel Dekumpe
K.U. Leuven, Campus Kortrijk (Belgium)

Polycyclic groups are built from cyclic (= easy) groups and thus, they form a class of groups which is relatively easy to understand. Nevertheless, a lot of interesting theory of these groups has been developed, and the theory of polynomial structures is one of those theories.

Originally one was interested in affine structures:

- **Affine structure on manifolds**: A manifold equipped with an atlas, having the property that transition functions between charts are affine maps of $\mathbb{R}^m$, are called affine manifolds (and they inherit the usual affine connection of $\mathbb{R}^m$).

Such a manifold is called complete if every geodesic can be defined on the whole time - axis $\mathbb{R}$. It is known that these manifolds are of the form $\mathbb{E}\mathbb{V}^m$, where $\mathbb{E}$ is acting properly discontinuously on $\mathbb{R}^m$ via affine actions. Therefore one is interested in:

- **Affine structure on groups**: An affine structure on a group $\mathbb{E}$ is a representation $\mathbb{S}: \mathbb{E} \to \text{Aff}(\mathbb{R}^m)$. Letting $\mathbb{E}$ act properly discontinuously and with compact quotient on $\mathbb{R}^m$, Auslander's conjecture states that all such $\mathbb{E}$ are polycyclic-by-finite. As a consequence of this, D. Segal questioned the existence of an affine structure on any polycyclic-by-finite group. In 1992 Y. Benoist produced an example of a group, not having an affine structure, therefore we initiated the investigation of polynomial structures on groups.
Gemäss und Topologie klassifizierender Braut für Familien


Laut [L-Dehnung 99] sei G die Kompakteneinkle Gruppe G/\Gamma,
und \Gamma sei ihre Modularisierung. Sei \Gamma \cap \Gamma(a) = \Gamma und die Familie der
kompakt-en Untern von G und \Gamma(a) dieselbe Menge wie
G/\Gamma. Dann ist es zusammenhang mit \Gamma\Gamma(a) bestimmt,

\dim (E(G,\Gamma(a))) \leq d \Rightarrow \dim (E(G,\Gamma) \leq d \Rightarrow
\dim E(G,\Gamma(a)) \leq d \Rightarrow \dim \varGamma(a) \leq \varGamma(a)) \leq 2

wobei \dim \varGamma(a) \leq 2 die homologische Dimension
der disvervanten Modelle 2 über der Orbitkategorie A(\Gamma(a),\Gamma) ist

Laut [L-Virtuell 79] ist die virtuelle Klassifikation diehnt. Sei \Gamma(a) \cap \Gamma(a) = \Gamma, \Gamma(\Gamma) \cap \Gamma(\Gamma) = \Gamma(\Gamma)
\dim \Gamma(a) \leq d \Rightarrow \dim E(\Gamma,\Gamma(a)) \leq d + \Gamma(a)

Vermutung [Braun 79] G virtuell Klassifikation: \dim \Gamma(a) \leq d \Rightarrow \dim E(\Gamma,\Gamma(a)) \leq d

Wolfgang Lück
Quasi-symmetric functions and the Linnaei-Hopf algebra

Michael Hazewinkel
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Let \( Z \) denote the Linnaei-Hopf algebra, which is also the shuffle descent algebra and the algebra of noncommutative symmetric functions.

As an algebra, \( Z = Z < z_1, z_2, \ldots > \), the free associative algebra over the integers in countably many indeterminates.

The algebra structure is given by

\[
\mu(Z_n) = Z \otimes Z^n, \quad Z_0 = 1
\]

let \( \omega \) be the graded dual of \( Z \). This is the algebra of quasi-symmetric functions. \( f(x) \) is quasi-symmetric iff to the ordering of indeterminates \( x_1 < x_2 < \ldots \) it applies all \( j_1 \ldots < j_n \), \( k_1 < k_2 < \ldots k_n \), and a nondegenerate sequence \( i_1, i_2, \ldots \) in the coefficient in \( f \). \( x_1^{i_1} x_2^{i_2} \ldots x_n^{i_n} \) is the same as that of \( x_1^{k_1} x_2^{k_2} \ldots x_n^{k_n} \). For example \( \omega(x_1^{k_1} x_2^{k_2} x_3^{k_3}) \) is not quasi-symmetric in \( x_1, x_2, x_3 \).

The Druthen conjecture states that \( \omega \) is a free commutative algebra over \( Z \).

I (1972) discuss the Druthen conjecture and give an outline of the second proof.

References:
M. Hazewinkel, The algebra of quasi-symmetric functions is free over the integers, preprint, March 1999.
Mapping Class Groups of Punctured Tori

John R. Parker (Durham)

The mapping class group (orientation preserving) \( \text{MCG}(\Sigma) \) of a surface \( \Sigma \) of finite type is the group of all (orientation preserving) homeomorphisms of \( \Sigma \) to itself up to isotopy.

A celebrated theorem of Dehn states that \( \text{MCG}(\Sigma) \) is finitely generated by certain elementary automorphisms, called Dehn twists about certain simple closed curves on \( \Sigma \).

Thurston showed that \( \text{MCG}(\Sigma) \) acts faithfully on PML(\( \Sigma \)), projective measured lamination space, the so-called Thurston boundary of Teichmüller space. Birman and Hilden showed how to give PML(\( \Sigma \)) a piecewise linear structure (using \( \pi_1 \)-train tracks) so that \( \text{MCG}(\Sigma) \) acts piecewise linearly on PML(\( \Sigma \)). Our aim is to describe this explicitly for the once and twice punctured tori. Applications include giving \( \text{MCG}(\Sigma) \) a concrete automorphic structure (known to exist by work of Mosher) or producing an algorithm to decide whether an element of \( \text{MCG}(\Sigma) \) is \( \partial \)-periodic, \( \partial \)-reducible or \( \partial \)-pseudo-Anosov (following ideas of Hain and Teichmuller).

For the once punctured torus, one obtains that any geodesic lamination may be written as a projective linear combination of a pair of elementary curves written symbolically as \( \Pi, \Pi, \Pi, \Pi, \Pi \) corresponding to lamination \( \{ -\infty, \infty \}, \{ -1, 0 \}, \{ 0, 1 \}, \{ 1, \infty \} \) of \( \mathbb{R} = \mathbb{R}_0 \times \mathbb{R}_\infty = \text{PML}(\Sigma, \partial) \).

\( \text{MCG}(1,1) \) is generated by Dehn twists \( S_0, S_1 \) about \( \Pi, \Pi, \Pi \) and has presentation \( \langle S_0, S_1 | S_0 S_1 = S_1 S_0 \rangle \). Putting \( S_0 = (1,1) \), \( S_1 = (1,1) \) we obtain the well known result that \( \text{MCG}(\mathbb{R}^2) = \mathbb{Z} \times \mathbb{Z} \).

For the twice punctured torus this may be generalised to decompose the 3-sphere (PML(\( \Sigma, \partial \))) into 28 tetrahedra obtained by putting \( \mathbb{R}^2 \) at the north pole, \( \mathbb{R}_\infty \) at the south pole and triangulating the symplectic plane as indicated.

\( \text{MCG}(\Sigma, \partial) \) is generated by Dehn twists \( S_0, S_1, S_2 \) about \( \Pi, \Pi, \Pi \) and has the following presentation:

\( \langle S_0, S_1, S_2 | S_0 S_1 = S_1 S_0, S_1 S_2 = S_2 S_1, S_2 S_0 = S_0 S_2, (S_0 S_1 S_2)^4 = e \rangle \).

\[ \mathbf{R.R.} \]

30 April 1999
7. 5. 99

A. Sänlin, Northton;

Polynomial factors and cohomology of algebraic and trivial groups.
Finite homotopy types and computation

Graham Ellis (NUI Galway)

This talk describes some computer calculations of 3-dimensional presentations of finite groups of low order. (Such a presentation is equivalent to the 3-skeleton of an Eilenberg-Marlow space \( X = K(\mathbb{Z},1) \).) The method involves applying the LLL algorithm (for finding bases of integer lattices) to the cellular chain complex of the universal cover of \( X \).

This naive approach does not work well if \( G \) is large. However, many large \( G \) arise from group-theoretic constructions applied to smaller groups. We describe some results on 3-dimensional presentations of such constructions. In particular, we give a result on the 3-presentation of a semi-direct product \( G \rtimes N \rtimes H \).

Our interest in 3-presentation arises from a desire to work towards an explicit enumeration of homotopy types \( X \) of "low order". We define the "order" of \( X \) to be

\[ |X| = \prod_{i \geq 1} |\pi_i X| \]

If \( |X| < \infty \) then clearly \( \pi_i X = 0 \) for all \( i \geq k+1 \), some \( k \).

Such an \( X \) is said to be a "homotopy \( k \)-type". Letting

\[ N(k,m) = \text{number of homotopy } k \text{-types of order } m, \]

the formula

\[ N(k, p^m) = \left\lfloor \frac{(p^{k+1})^m}{(p^{k+1})^k - 1} \right\rfloor \]

will be explained in the talk.

G. Ellis

14.5.95
The distribution of algebraic numbers

The set of rational numbers is dense in \( \mathbb{R} \), but the neighborhood of the rational number \( \frac{a}{b} \) with a small denominator does not contain other rational numbers with small denominators. Therefore any sequence of rational numbers with natural ordering can’t be "good" uniform distributed in the sense of good upper bound for the. It is natural to define the new notion of distribution of algebraic numbers. This definition was given by A. Baker and W. Schmidt in 1970. They called it the regular system (RS) and proved that the set of algebraic numbers forms RS: for any interval \( I = [a, b] \subset \mathbb{R} \) there exists \( K_0(\epsilon) \) such that for every \( K > K_0 \), we can find real algebraic numbers \( \alpha_1, \alpha_2, \ldots, \alpha_t \in I \) such that

\[
H^{\frac{n+1}{n+2}}(\alpha_j) \log \frac{1}{H(\alpha_j)} < K, \quad 1 \leq j \leq t,
\]

where \( H(\alpha_j) \) is the height of the algebraic number \( \alpha_j \), with \( y = \frac{3n}{n + 1} \), and the properties

1) \( |\alpha_i - \alpha_j| > K^{-1}, \quad i \neq j \); 2) \( t > c(\epsilon)(b-a)K \)

In 30-th this result was improved \( (y=2) \) and recently V. Beresnevich obtained \( y=0 \). These results have a lot of applications in diophantine approximation. In 1938 D. Vasilyev and V.B. constructed best possible RS of algebraic complex numbers. Both results allow to prove a full analogue of the classical theorem of Khintchine about exact order of approximation of almost all real numbers by the rational numbers.

V. Bernik, Institute of Mathematics of Academy of Sciences (Minsk).
28.05.95

Brauer groups and u-invariant of some fields.

The u-invariant of a field \( F \) is defined as

\[ u(F) = \sup \{ d \mid q \text{ is an anisotropic quadratic form over } F \text{ such that } \Delta(q) \text{ is a natural homomorphism} \} , \]

where \( \Delta(q) \) is a natural homomorphism.

\[ \Phi \colon W(F) \to \Omega W(F) \]

where \( \Omega \) is the set of all orderings of \( F \), \( F_0 \)

a real closure of \( F \) with respect to \( \Omega \) \( W(F) \) (resp. \( W(F) \)) Witt groups of \( F \) (resp. \( W(F) \))

and \( \Phi(\Omega) \) is the element of \( W(F) \) corresponding
to \( \Phi \).

In connection with one of Lang's conjectures related to non-formally real function fields (let \( F \) be a non-formally real field of transcendence degree 1 over real closed field, then it is a \( C_2 \)-field). A. Pfister conjectured in 1982 that if one has a field \( F \) of transcendence degree at most 2, with real closed, then \( \Phi(\Omega) \leq 4 \).

The local variant of this problem for function fields is as follows:

Conjecture. Let \( F \) be a function field of smooth projective curve \( C \) defined over the field \( \mathbb{R}(\{t\}) \) of formal power series with over real closed field \( \mathbb{R} \). Then \( \Phi(\Omega) \leq 4 \).

Remark. Note that in case \( F \) is non-real.

It is a conjecture of Lang for \( i = 2 \).

We discuss some aspects of positive solutions of the conjecture above by using computations in Brauer groups of hyperelliptic curves defined over \( \mathbb{R}(\{t\}) \).
Finiteness properties of arithmetic groups

or: Reduction, filtration, retraction

Helmut Behr (Frankfurt a.M.)

1) Development of reduction theory
   - of quadratic forms
   - for arithmetic groups
   - with compactification (Bogel '59, Borel-Serre '78)
   - with "canonical" filtration (Auclet, Stabler, Pajer, '72-84)
   and retraction to the boundary of the unstable region.

2) (Hipsch) finiteness properties of groups: FP_n and F
   a) All finitely generated subgroups of reductive algebraic
groups over a number field are of type FP_n (Borel-Serre '78)
   b) Over function fields, there exist many counter-examples,
      and also series of groups with type F, not FP_n,
      (results of Milne, Abels, Abramenko, Behr, etc.)
   c) Let G be an alg. group, defined over F, [F: F_q(t)] < oo
      almost simple of rank r over F, S a finite set of primes
      of F, r_v the rank of G over the completion F_v of F for v \in S
      a finitistic subgroup of G

Question: Is \Gamma of type F_{n-1}, not FP_n \iff r > 0, \sum_v r_v = n.

For all results in b, this is true -- for some results, the assumption
that q is big enough with respect to n.

Program for a proof: Let X be the Bruhat-Tits building \Gamma (G)
X is the building at infinity, X^1 the unstable region and
define a "space of half-lines"
Z = \{ (x, t) : t \in \mathbb{R}^+ \} by the opposition relation for X^0
Z has a covering, where each is \Gamma opp X, which is
n-spherical at \infty; \Gamma retracts to the (infinite) boundary \partial Z; \Gamma/\text{finite}

Prop: G almost simple Chevalley, F of rank r, then
   \Gamma = G(F_q[t]) is of type F_{r-1}, not FP_r.

 Conj: This is also true for \Gamma finitistic if \# S = 1.

- H. Behr
Isotropy of quadratic forms over function fields of quadrics

Oleg Izhboldin (St. Petersburg / Bielefeld)

Let $F$ be a field of characteristic $\neq 2$, let $\varphi$ and $\psi$ be (non-degenerated) quadratic forms over $F$. We define the order relation $\varphi \leq \psi$ as follows:

1. $\varphi \leq \psi$ if and only if for any field extension $L/F$ we have:
   - if $\varphi$ is isotropic over $L$ then $\psi$ is isotropic over $L$
   - $\varphi \leq \psi$ if and only if for any field extension $L/F$ we have:
     - $\varphi$ is isotropic over $L \iff \psi$ is isotropic over $L$.

Remark 1. $\varphi \leq \psi$ if and only if the form $\varphi$ is isotropic over the function field of $\psi$. It is only if there exists a rational morphism of quadric $X_\varphi$ to the quadric $X_\psi$.

1. $\varphi \leq \psi$ if and only if $X_\varphi$ and $X_\psi$ are stably birationally isomorphic.

Examples:

a) if $\psi$ is isotropic then $\varphi \leq \psi$
   1. if $\psi$ is similar to a subform of $\varphi$ then $\varphi \leq \psi$
   2. if $\psi$ is a Pfister neighbour of a Pfister form $\varphi$ then $\varphi \leq \psi$
   3. if $\psi = \psi_1 \cdots \psi_n$ and $\varphi$ is a subform of $\psi_1 \cdots \psi_n$ of codimension $\geq \frac{1}{2} \dim \psi$ then $\varphi \leq \psi$
   4. if $\psi = \psi_1 \cdots \psi_n$ (dim $\psi_1 \geq 2$) and $\varphi$ is a subform of $\psi$ of codimension $\geq 2$ then $\varphi \leq \psi$

Definition. Let $\varphi$ and $\psi$ be such that $\varphi \leq \psi$. We say that the relation $\varphi \leq \psi$ is standard if there exist $q_0, q_1, \ldots, q_k$ such that $q_0 \leq q_1 \leq \cdots \leq q_k \leq q_n$ and $q_0 = \varphi_0, q_1 = \varphi_1, \ldots, q_k = \varphi_k$, and the relation $q_0 \leq q_k$ holds to list of Examples 0-4.

Theorem 1. Let $\varphi$ and $\psi$ be such that $\varphi \leq \psi$. Then "$\varphi \leq \psi$ is standard" in the following cases:

1. $\varphi = \psi$ is a Pfister neighbour
   2. $\dim \varphi \leq 5$
   3. $\dim \varphi = 6$, except for the case where\[ \dim \psi = 4 \quad \text{and} \quad \text{det} \psi = \text{det} \varphi + 1 \quad \text{and} \quad \text{ind} \varphi = \text{ind} \psi = 2n \quad \text{and} \quad 2^n - 1 \]
4. $\dim \psi = 3$, $\psi \in \text{E}_6$, $\dim \varphi \geq 5$
5. $\dim \psi \leq 2^n$ and $\dim \varphi \geq 2^{n+1}$
6. $\dim \psi = 2^{n+1}$ and $\dim \varphi \geq 2^{n+1}$

Theorem 2. There exists a "most standard relation $\varphi \leq \psi$" with $\dim \psi = 6$.

The proofs of Theorem 1 could use methods of algebraic $K$-theory, Galois cohomology on $\text{CH}^n$-groups of quadrics (and products of quadrics).

Over und dernde, [Name]
An excursion into analysis and irregularity

Umberto Mosco (Roma)

Analysis has been confronted with "irregularities" of various kind all along the century.
"Irregularity" is used here in a broad sense, referring to analytic behavior deviating in a substantial way from classic Euclidean or Riemannian models.

A good insight into a large class of non-Euclidean structures can be obtained by focussing on basic concepts like length, volume, energy and on the mutual relations existing among them. The analytic tools for such a general theory can be found in the fundamental works of De Giorgi and Moser on uniformly elliptic operators in divergence form and measurable coefficients, of John - Nirenberg on $BMO$ spaces of functions with bounded mean oscillation and in Hörmander's hypoellipticity theory for smooth vector fields and related stratified Lie groups, like the Heisenberg group and in further developments from a metric point of view by Rothschild - Stein - Fefferman-Phong, Nagel-Stein-Wainger, Jerison and others, in the last two decades.

The conclusion we can draw by analysing these theories closely, as well as the recent mathematical and physical work on fractals, can be summarized as follows:

Various classical, semi-classical and fractal theories of "elastic bodies" possibly of very irregular nature can be built on a common metric background,
involving locally compact topological spaces $X$ and
- quasi-distances
  \[ d(x, y) \leq c \left[ d(x, z) + d(z, y) \right], \quad c > 1 \]
- doubling measure
  \[ \mu(B_R) > c \cdot \mu(B_{R/2})^{2^d}, \quad R > 0, \]
  \[ B = B_R, \quad 0 < r < R \]
- measure-valued gradient forms ("Lagrangians")
  \[ L(u, v) = \nabla u \cdot \nabla v \]
defined on dense subalgebras $C$ of $\mathcal{C}(X)$, all of them related mutually by the property:
Lagrangians control bounded mean oscillations at all metric scales:

\[ \int_{B^q} |\bar{P}_B|^{1/q} \, d\mu \leq c \cdot R \left( \int \left| \frac{\partial L(u, u)}{\partial z} \right|^q \, dz \right)^{1/q}, \quad u \in C \]

\[ \frac{f}{B} = \mu(B)^{-1} \int_B f \, d\mu, \quad \bar{f}_B = \sup_{B_R^q} f \, d\mu, \quad B = B_R, \quad q > 1. \]

All previous properties taken together give rise to $X$, $d$, $\mu$, $L$ the notion of a "pseudo-Riemannian" space. The main analytic procedures leading to these structures are Lie-group invariance in the smooth case and self-similarity invariance in the fractal case.

[Viola, 1954]
Complex geometry of real symmetric spaces.
Simon Gindikin (Rutgers University & MPI)

Let $G_o / K_o$ be a real symmetric space, $X = G / K$ be its complexification. There is a canonical Stein manifold $X_o$ satisfying to $\circ$ the condition: all zonal spherical functions on $X_o$ admit holomorphic extensions on $X$ and it is a maximal manifold with such a condition. We call $Y$ the crown of $X_o$: $Y = \text{Crown}(X_o)$. Many other objects admit homomorphic extension on $Y$.

Eigen function of Laplace operator, solutions of Schródinger equations (in which discrete series of representations can be realized), kernels of Szego operators etc.

There are several geometric conjectures which are checked only at some examples. Crowns are $G$-orbit invariant, but not homogeneous. There is a hypothetical conjecture on $\circ$ the parametrization of $G_o$-orbits. There is a conjecture that $Y$ gives a parametrization of complex cycles at flog domains. It's proved for groups of Hermitian type and for $SL(n, \mathbb{R})$, $SU(n)$ and a few examples.

The second subject of the talk - tube domains which edges are pseudo-Riemannian symmetric spaces. The conjecture is that it is possible to define Hardy spaces of $\Delta$-cohomology in which different series of representations can be realized between these tubes some are Stein manifolds which correspond either holomorphic discrete series or maximal constant-times series on causal symmetric space. In the case least left tubes coincide with crowns of corresponding Riemannian symmetric spaces.

Simon Gindikin
25.06.99

Restrict Maps between Cohomologies of Arithmetic Groups
T. N. Venkataramana

(School of Math., Tata Institute of Fundamental Research, Homi
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We establish a criterion to determine when a cohomology
class \( \omega \in H^*(S\Gamma, \mathbb{C}) \), of a smooth compact locally
Hermitean symmetric variety \( S\Gamma \) (\( \Gamma \) is a torsion-free
arithmetic subgroup of the group \( G \) of holomorphic
automorphisms of a Hermitean symmetric domain of
noncompact type), “restricts” non-trivially to a smaller
complex submanifold \( S_H(\Gamma) \) of \( S\Gamma \) (here \( S_H(\Gamma) \) is
also a compact locally symmetric variety). Here, the
“restriction” is in the sense that one may have
to move the class \( \omega \) by a Hecke correspondence
and then to restrict the resulting class to \( S_H(\Gamma) \).

Let \( \hat{X} \) and \( \hat{Y} \) be the compact dual symmetric spaces
of \( S\Gamma \) and \( S_H(\Gamma) \), respectively; let \([\hat{Y}]\) be the Poincaré
dual of \( \hat{Y} \) in \( H^*(\hat{X}) \). Via the Matsushima Formula,
\( H^*(\hat{X}) \) may be thought of as a subring of \( H^*(S\Gamma) \).
Let \([S_H(\Gamma)]\) denote the Poincaré dual of the special
cycle \( S_H(\Gamma) \) in \( S\Gamma \).

**Theorem 1:** 1) The class \([\hat{Y}]\) is a linear combination of
Hecke translates of \([S_H(\Gamma)]\).

2) If all the Hecke translates of a
class \( \omega \) restrict trivially to \( S_H(\Gamma) \), then \( \omega \cdot [\hat{Y}] = 0 \).

Using Theorem 1, we may show

**Theorem 2:** If \( X \) (resp. \( Y \)) is the unit ball in \( \mathbb{C}^n \)
(resp. the unit ball in \( \mathbb{C}^m \)), then all the cohomology
of $s(T)$ up to degrees $\leq \dim Y$, "restrict" injectively to $s_H(T)$. This affirms a conjecture of M. Harris and J.-S. Li.

Theorem 1 also implies the following:

**Theorem 3:** Let $s(T)$ be a quotient of the unit ball in $\mathbb{C}^n$, with non-trivial first Betti number, and let $Z = s(T)$ be a (closed) smooth subvariety. Then there exists a finite covering $\mathcal{Z}$ of $Z$ such that all its Hodge numbers $h^{p,q} = \dim \mathcal{H}^{p,q}(\mathcal{Z})$ are nonzero (for all $p, q \leq \dim Z$).

To prove Theorem 3, one notes that restricting a tensor product $\omega \otimes \omega' \in H^*(s(T) \times s(T'))$ (of classes $\omega$ and $\omega'$ on $H^*(s(T))$ to the diagonal $s(T)$, is merely taking the cup-product $\omega \cup \omega'$ of these classes. Theorem 3 is then shown to follow by using the criterion of Theorem 1.
Hardy theorem for some Lie groups

-- a version of uncertainty principle --

Keisuke KUMAHARA (The Univ. of the Air, Chiba)

The uncertainty principle says that a function is concentrated, then its Fourier transform cannot be concentrated unless it is identically zero. The most well-known theorem is the Heisenberg - Pauli - Weyl inequality. And many generalizations and variations are known. One of them is the Hardy theorem, which yields that if a measurable function $f$ on $\mathbb{R}$ satisfies

$$|f(x)| \leq C \exp(-ax^2), \quad |\hat{f}(\xi)| \leq C \exp(-b\xi^2)$$

for $C > 0$, $a > 0$, $b > 0$ and $ab > \frac{1}{4}$, then $f = 0$ (a.e.). Here we take $\hat{f}(\xi) = (2\pi)^{-\frac{1}{2}} \int_{\mathbb{R}} f(x) e^{-i\xi x} \, dx$ as the definition of the Fourier transform. M. Cowling and J. F. Price an $L^p$ version of the Hardy theorem (1983): If a measurable function $f$ on $\mathbb{R}$ satisfies $\|e^{ax^2}\|_{L^p} < \infty$, $\|e^{b\xi^2}\|_{L^q} < \infty$ for some $p, q$ which are $1 \leq p, q \leq \infty$ and one of them is finite and $ab \geq \frac{1}{4}$, then $f = 0$ (a.e.)

In this talk, I report some generalizations of such theorems.

(1) an analogue of the Hardy theorem for Cartan motion group
(2) an analogue of the Hardy theorem for connected noncompact semisimple Lie groups
(3) an $L^p$ version of the Hardy theorem for the motion group

(1) and (3) are joint works with M. Eguchi and S. Koizumi, (3) is joint work with M. Ebata, M. Eguchi and S. Koizumi.

Keisuke KUMAHARA

熊原 瑠作
CURVATURE NOTIONS IN GROUP THEORY
(JENS HARLANDER, UNIV. FRANKFURT)

METRIC AND GEOMETRIC STRUCTURES ON CELL
COMPLEXES HAVE A LONG HISTORY, POSSIBLY
BEGINNING WITH DENN'S ALGORITHM FOR
SOLVING THE WORD PROBLEM IN INFINITE
GROUPS. IN GENERAL, GIVEN A CW-
COMPLEX, ONE CAN METRIZE THE INDIVIDUAL
CELLS AND THEN EXTEND THE LOCAL METRIC
ON THE WHOLE COMPLEX. CURVATURE-
CONSIDERATIONS FOR METRIC SPACES APPEAR
FIRST IN THE FUNDAMENTAL WORK OF
ALEXANDROV. THESE NOTIONS WERE RE-
INTRODUCED TO MAIN STREAM GEOMETRIC
GROUP THEORY BY GROUPE IN CONNECTION
WTH UNPERIODIC GROUPS. IN MANY
CASES, COMPLEXES NONTRIVIAL OR
CONTINUATION PASSING THAT ESTABLISH
HOMOLOGY CAN BE CONSIDERABLY
SIMPLIFIED USING GEOMETRIC TECHNIQUES.

IN MY TALK I WILL DISCUSS METRIC
AND NON-METRIC ASYMPTOTIC TESTS
BASED ON CURVATURE NOTIONS.
I WILL ILLUSTRATE THE TECHNIQUES BY
GIVING APPLICATIONS TO KNAP AND
COXETER GROUPS, KNOT AND RIBBON-DISC
COMPLEMENTS.
Nichtkommutative Charaktere: ein neuer Zugang zur Charaktertheorie der
gemessenen Gruppen.

Die Charaktertheorie der gemessenen Gruppen läßt sich elegant
und übersichtlich organisieren mit Hilfe der Brücke $\bigoplus C$ der
Klassenfunktionen, wie Giraud 1977 gezeigt hat. Im einzelnen
bedeutet dies: Wir setzen $C = \bigoplus C(S_n)$ — die direkte Summe
aller Vektorräume der $k$-weltigen Klassenfunktionen auf den
gemessenen Gruppen $S_n$. $C$ ist auf beliebiger Multiplikation
eine Algebra $(C, \cdot)$, Tensorprodukt und Induktion liefern eine
weitere Multiplikation $\triangleright$ auf $C$. Ein Coprodukt $\triangleright: C \to C \otimes C$
ergibt sich durch Reduktion der Klassenfunktionen auf geeignete
Young-Untergruppen (parallelepipedische Untergruppen). Die üblichen
Bilinearformen auf den Räumen $C(S_n)$ setzen man in eine
Bilinearform auf $C$ zusammen. Die klassische Charaktertheorie ist
damit die Theorie der Räume in $(C, \cdot, \triangleright, \triangleright, (,))$, wobei das
Rechnen mit irreduziblen Charakteren das Hauptproblem ist.

Ein neuer Zugang wird durch die Konstruktion einer geeigneten
$\oplus$-algebra mit Skalarprodukt $(\oplus, \cdot, \oplus, (,))_{\oplus}$ und einer
Algebra epimorphismus $c:(\oplus, \cdot) \rightarrow (C, \cdot)$ gegeben, für den
außerdem $c(q) \triangleright = (c \otimes c)(\triangleright q)$ und $(q, q)_{\oplus} = (c(q), c(q))_{\oplus}$
gilt. $\oplus$ wird ergänzt von den Summen $2^{1, 3, q}$ aller Standard-Young
Tableaux der jeweiligen Gestalt $\mu\nu$, wo $\mu\nu$ Partitionen sind.
Setzt man $2^h = \sum \chi_h (c) \in SYT$, wobei SYT die Menge aller
Standard-Young-Tableaux der Gestalt $\mu\nu$, $\mu\nu$ eine Partition ist, so ist
$\chi = c(2^h)$ der einzige irreduzible Charakter von $S_n$, dessen Bild unter
dem Faktor $c: \oplus \rightarrow C$ gerade die Schurfunktion $\nu \pi$ ist. Der Satz
von Littlewood - Richardson bedeutet jetzt in der Gestalt
$(\xi^h \otimes \xi^q, \xi^h) = (2^{1, 3, q}) \xi^h (2^{1, 3, q})$,
eine Form dieses Satzes die 1985 von Garsia und Remmel ausgegeben
wurde. Auch die anderen Sätze der klassischen Charaktertheorie lassen mit
Hilfe dieser nicht-kommutativen Überbegriffe $C$ leicht liefern.
Solchems deesr $\oplus$-algebra ist enthalten in $\oplus$ und $c$ eine Fortschritt
Dynamical Systems and Controllability: A Systems Theory View

This talk proposes some mathematical concepts for 'open' dynamical systems. We start with a specific example, the problem of obtaining a mathematical model for a simple RLC circuit. This leads to the problem of how to calibrate the resulting 'first principles' model into mathematics. After a brief historical remarks, we end up with a definition in terms of a behavior defined by equations that involve both manifest and latent variables. We subsequently restrict attention to linear shift-invariant differential systems.

Three specific problems are discussed: 1. the elimination problem 2. controllability, and 3. observability. The elimination theorem basically states that the behaviors of linear shift-invariant differential systems are closed under (intersection, addition, and) projection. The next topic addressed is controllability, which is defined as a 'patching' property. We show that controllability is equivalent to the existence of an image representation, i.e., of a potential function. We finally discuss, very briefly, the 'dual' notion of observability.

As closing remarks, we allude to some applications of these ideas in control and filtering, coding (convolutional codes), to first-order representations, and to the theory of dissipative structures.

Jan C. Willems, Univ. of Groningen, NL

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Die Untersuchung der globalen Dynamik ebener (und erst recht höher dimensionaler) polynomialer Vektorfelder entbehrt bisher einer festen und einheitlichen Grundlage – wie etwa die algebraische Geometrie besser Commutative Algebra “erfahren” wurde.

Im Vortrag soll eine neuartige Methode der “Pseudo-orbit im Pseudophasenraum” bzw. der “Pseudo-potentiale” vorgestellt werden, welche sich am Bsp. einer gewissen Klasse ebener polynomialer Vektorfelder bewährt hat.

Für solche ebener Vektorfelder nämlich, welche sich durch die Steifigkeit komplex-eindimensionaler Vektorpolynomialer Vektorfelder aller Art ergibt, bietet die neue Methode u.a. folgende Ergebnisse:  
1) Lokale und globale Fragen können auf eine gemeinsame Grundlage behandelt werden,
2) Nicht-existenz von Grenzzyklen,
3) Völligstehende kombinatorische Beschreibung aller möglichen topologischen Typen von Flüssen,
4) Nachweis, dass alle kombinatorischen Typen auch tatsächlich durch Flüsse realisiert sind, also vollständige topologische Klassifikation,
5) Eine neue Familie globaler Bifurkationen tritt auf,

etc., etc.
Ein neuerige Form von point plots
für die Löungsmethoden der lokalen
Deterninen-Längen soll derart gestellt werden,
mit der man erkennt, dass eine wichtige
Voraussetzung für die Anwendbarkeit der
Pseudo-arbeit-methode universell für
alle polynomialen Größen und Dimensionen
erfüllt ist. Diese point plots spielen
in Funktion (noch nicht in ihrer Form) die
gleiche Rolle wie andere mero in der Iteration
polynomialer Abbildungen.

Dr. Rudolf Winter
RWTH - Aachen

12.11.99

Generische Initialideale und graduierter
Betti Zahlen.

In diesem Vortrag werden Sätze von
G. Kalai über "algebraische Stufung",
hergeleiten. Zu jedem gegebenen
Simplicialen Komplex \( \Delta \) definiert Kalai
den "geschäfteten" Komplex \( \Delta^\epsilon \) wie folgt:
Man betrachtet in der äußeren Algebra
der Stanley-Reisner Ideale \( \Delta^\epsilon \).
Dann ist $A^c$ (der geschliffene Körper) definiert durch die Gleichung $T_A^c = G_{\text{Rl}(A)}$. 

Jürgen Herzog

Univ. Essen
Let $k$ be a field of characteristic zero, and $\text{GL}_n := \text{GL}_n(k)$.

We embed $\text{GL}_{n-1}$ into $\text{GL}_n$ by the map $t \mapsto (t \ 1)$. Then denoting by $B_n$ the Borel subgroup of all upper triangular matrices in $\text{GL}_n$, $B_{n-1}$ acts on the flag variety $\text{GL}_n/B_n$ with an open dense orbit. In this talk I give an algorithm to determine all the $B_{n-1}$-orbits, and describe the "Bruhat order", i.e. the closure relation between the orbits in a fixed Bruhat cell. As a corollary, one finds the orbits whose closures coincide with the Schubert varieties.

Takashi Hashimoto
(Tottori Univ. / Strasbourg Univ.)
3 December 1999

Units in Integral Group Rings

The integral group ring \( ZG \) of a group \( G \) is a natural object where group- and ring theory meet. Quite recently, M. Herbig gave a counterexample to the old isomorphism problem, that is, the question if two non-isomorphic groups \( G \) and \( H \) so that \( ZG \) and \( ZH \) are isomorphic. This again gave a boost to the study of the unit group of an integral group ring.

Very recently, Moszur, Janssen and Jespers showed that there is a close relationship between the isomorphism problem and the way the group \( G \) is embedded in \( U/ZG \), the units. In particular, the isomorphism problem holds for \( G \times A \), with \( A \) a finitely generated free abelian group, if and only if the isomorphism problem on abelian groups holds for \( G \). That is, the isomorphism of \( G \times U/ZG \) and \( U/ZG \) is equivalent to \( G \) being isomorphic to the group of units of \( U/ZG \) (the trivial units being the abelian units). We will survey some of these results.

Also we survey some of the recent results on the big lurking in the isomorphism of \( U/ZG \), \( G \) a finite group, namely the structure theorem of the unit group.

For most finite groups \( G \) we know a finite set of constructive generators of a subgroup of finite index in \( U/ZG \) but we lack knowledge on the structure. However, one can clarify the finite groups \( G \) so that \( U/ZG \) contains a subgroup of finite index that is a free product of abelian groups. The latter also needs much del Rios and Leal.

Eric Jespers
Vrije Universiteit Brussel
Belgium
Let $G$ be a connected, reductive, linear algebraic group over $\mathbb{C}$ and let $\sigma$ be an involutorial automorphism of $G$, with fixed point group $H$. The quotient $G/H$ is a symmetric variety. A Borel group $B$ of $G$ has finitely many orbits on $G/H$ (i.e. the number of double cosets $BgH$ is finite). Let $V$ be the set of these orbits. It has a partial order, induced by inclusion of orbit closures.

In joint work with R.W. Richardson (Geom. Dedic. 35 (1990), 389-436) a combinatorial set-up was introduced, in order to study the ordered set $V$. It turns out that this set has some resemblance to the Weyl group of $G$, with its ‘Bruhat order’.

The symmetric variety $G/H$ has a ‘wonderful’ compactification $X$, introduced by De Concini and Procesi in 1983. It is a smooth, projective, $G$-variety containing $G/H$ as an open subvariety. $B$ has finitely many orbits on $X$, let $\tilde{V}$ be the ordered set of these orbits.

In the talk I discussed the extension to $\tilde{V}$ of the combinatorial set-up for $V$. The results are not yet as complete as one would wish.

T.A. Springer
Universiteit Utrecht
The talk is concerned with the probability of an incorrect decoding when an \( \ell_n, k, d_L \) code is used for error correction. If the channel has symbol error probability \( p \), the probability we are interested in is the conditional probability
\[
F(p, t; C) := P(\text{incorrect decoding} \mid d(\text{received word}, \text{decoded word}) \leq t)
\]
where \( t \leq d_L - 1 \) and \( d(., .) \) denotes the Hamming distance.

For small \( p \) and \( \ell_n, k, d_L \), codes \( C \) resp. \( C' \) with weight distribution \((A_0, \ldots, A_n)\) resp. \((A'_0, \ldots, A'_n)\), A. Feldman recently proved:

\[
F(p, t; C) < F(p, t; C') \quad \text{if and only if} \quad (A_0, \ldots, A_n) < (A'_0, \ldots, A'_n) \quad \text{in lexicographical order.}
\]

HDS codes have minimal decoding error probability in the class of all \( \ell_n, k, d_L \) codes.

We present a class of codes, so-called MHD codes, i.e.,
codes with weight distribution \((A_0, \ldots, A_d, 0, 0, \ldots, 0, A_n - d + 2, \ldots, A_n)\)
which have minimal decoding error probability in the class of all \( \ell_n, k, d_L \) codes. (Here \( d_L \) denotes the minimum distance of the dual code.) The classification of such MHD codes mainly relies on the rigid structure which the set of column vectors of a generator matrix has to satisfy.

Wolfgang Willems

Otto-von-Guericke Universität Magdeburg
Verallgemeinerte Standard-Tableaux in der Darstellungstheorie endlicher Gruppen

Nach dem Satz von Wedderburn ist die komplexe Gruppenalgebra einer endlichen Gruppe $G$ isomorph zu einer Algebra im Blockdiagonalmatrizen $D: C G \rightarrow \mathfrak{g}$, die derartige Algebrenisomorphismen heißt eine diskrete Fouriertransformation (DFT). Die $k$ Projektionen $D_1, \ldots, D_k$ von $D$ bilden ein vollständiges System paarweise inäquivalenter irreduzibler Darstellungen in $C G$. In dem Vortrag gehe ich folgenden Fragen nach:
- Wie kann man schnell irreduzible Darstellungen konstruieren?
- Welche DFTs lassen sich schnell auswerten?


Schließlich besprechen wir einige Anwendungen:
- schnelles Falteln in Gruppenobergruppen
- Kollektiv in p-Gruppen mittels DFTs
- Berechnung von Charaktertafeln.

Michael Clausen

(17.12.99)