ON RATIONAL POINTS OF LINEAR ALGEBRAIC GROUPS

R. Gille, 8/01/9

Let k be a field with characteristic 0. The R-equivalence on the group $G(k)$ of k-rational points of G is an obstruction to the parametrization of $G(k)$; in the case of tori, it is the only one. We remind the following conjecture: Is $G(k)/R$ abelian? For all known cases, $G(k)/R$ is abelian (eg $SL_1(D)$, $Spin(q)$, ...).

We discuss the following examples.

* Norm form, ie equation $N_{L/k}(x)=1$ for a field ext. L/k

* $SL(D)$ after Rost.

* $PSO(q)$ (after Merkurjev, talk of the 10.th december 93).

About the last example, we have the following additional result.

Theorem: There exists a field k, and a quadratic form q/k of rank 8 with discriminant 1 such that $PSO(q)(k)/R \neq 1$.

We discuss although the Manin's finiteness conjecture.

Conjecture: Let G/k be a linear algebraic group defined over a field k finitely generated over $\mathbb{Q}$. Then $G(k)/R$ is finite.

In the case of tori, this conjecture was proven by Colliot-Thélène–Sansuc and in the case of number fields, we have the:

Theorem. Let G/k be a linear algebraic group defined over a number field k. Then $G(k)/R$ is finite.

Gille

Exceptional sets in analysis and probability theory

E. B. Dynkin

January 15, 1999

It is well-known that a subset of $\mathbb{R}^d$ is not hit, a.s., by the Brownian motion if and only if its classical capacity is equal to 0. More recently, it was established that the sets not hit by the super-Brownian motion can be characterized similarly in terms of suitable Bessel capacities. These sets are also removable singularities for solutions of certain semilinear PDEs. The most recently, analogous connections were proved for boundary singularities. The theory is rather complete for domains with a smooth boundaries. There are still challenging open problems for general domains (or smooth manifolds) and the corresponding Martin boundaries.

Der Chern-Connes Charakter in der nichtkommutativen Geometrie.

J. Cuntz, 15.1.99

Für geeignete Kategorien $\mathcal{C}$ von Algebren über $\mathbb{C}$ (oder $\mathbb{R}$) können bivariante K-Theorie-Gruppen $kk_i(A,B)$ und bivariante (periodische) zyklische Homologiegruppen $HP_i(A,B)$, A, B in $\mathcal{C}$, $i \in \mathbb{Z}/2$ definiert werden. Diese Theorien verallgemeinern die klassische K-Theorie und de Rham Theorie und liefern gleichzeitig einen ganz neuartigen Zugang.

Wir beschreiben einen bivarianten multiplikativen „Chern-Connes-Charakter" $Ch: kk_i(A,B) \to HP_i(A,B)$ und diskutieren seine Eigenschaften.

J. Cuntz
Universität Münster

Homotopietheoretische Methoden
in der Kohomologietheorie arithmetischer Gruppen

Hans-Werner Henn 29.1.99

Der mod p Kohomologiering einer (S-) arithmetischen Gruppe ist nur in sehr wenigen Fällen explizit bekannt, für $SL(n, \mathbb{Z})$ z.B. nur für $n \leq 3$. Wir beschreiben Fortschritte im Verständnis der Kohomologie S-arithmetischer Gruppen (z.B. $SL(n, \mathbb{Z}[\frac{1}{2}])$, $O_n(\mathbb{Z}[\frac{1}{2}])$), die mit modernen Methoden aus der Homotopietheorie klassifizierender Räume erzielt worden sind.

Hans-Werner Henn

Strasbourg

Gauß periods in finite fields

Joachim von zur Gathen
Paderborn

Carl Friedrich Gauß invented his periods for his construction of the regular 17-gon. Taken modulo prime numbers (or prime ideals), these Gauß periods sometimes provide normal elements in finite field extensions. We present two applications. The first one is to find elements in finite fields of exponentially large order with an efficient algorithm; this is a step towards the open problem of finding primitive elements efficiently. The second application is to fast exponentiation. For fields like $\mathbb{F}_{2^n}$, this yields algorithms that are fast both in theory and in practice. Finally, we present a more general construction than Gauß' original one, which gives a wider range of applicability.

5 February 1999

Polynomial Structures on Polycyclic Groups.

12 February 1999

Karel Dekimpe
K.U.Leuven, Campus Kortrijk (Belgium)

Polycyclic groups are built from cyclic (= easy) groups and thus, they form a class of groups which is relatively easy to understand. Nevertheless, a lot of interesting theory for these groups has been developed, and the theory of polynomial structures is one of ~~these~~ theories.

Originally one was interested in affine structures:

- Affine structures on manifolds: A manifold equipped with an atlas, having the property that transition functions between maps are affine maps of $\mathbb{R}^n$, are called affine manifolds (and they inherit the usual affine connection of $\mathbb{R}^n$) Such a manifold is called complete if every geodesic can be defined on the whole time-axes $\mathbb{R}$. It is known that these manifolds are of the form $_E\backslash\mathbb{R}^n$, where E is acting properly discontinuously on $\mathbb{R}^n$ via affine motions. Therefore one is interested in:
- Affine structures on groups. An affine structure on a group E is a representation
$$\rho: E \to \mathrm{Aff}(\mathbb{R}^n)$$
letting E act properly discontinuously and with compact quotient on $\mathbb{R}^n$. Auslander's conjecture states that all such gps are polycyclic-by-finite. As a converse of this, Milnor questioned the existence of ~~of~~ an affine structure on any pol-by-finite group. In 1992 Y. Benoist produced an example of a pol-by-finite group, not having an affine structure, therefore we started the investigation of
- Polynomial structures on groups. (i.e. we replaced $\mathrm{Aff}(\mathbb{R}^n)$, by $P(\mathbb{R}^n)$, the group of polynomial diffeomorphisms of $\mathbb{R}^n$)

Geometrie und Topologie klassifizierender Räume für Familien

Sei G eine lokal-kompakte topologische Gruppe. Sei $\mathcal{F}$ eine Familie von Untergruppen, d.h. eine Menge von (abgeschlossenen) Untergruppen, die unter Konjugation und Durchschnitt abgeschlossen ist. Sei $E(G,\mathcal{F})$ der zugehörige klassifizierende Raum, ein G-CW-Komplex mit $\mathrm{Iso}(X) \subset \mathcal{F}$, für den $E(G,\mathcal{F})^H \simeq_w *$ für $H \in \mathcal{F}$ gilt. Er hat die universelle Eigenschaft, daß es zu jedem G-CW-Komplex X bis auf G-Homotopie genau eine G-Abbildung $X \to E(G,\mathcal{F})$ gibt. Wir beschäftigen uns mit der Frage, wann es ein d-dimensionales Modell gibt. Neben Sätzen von Kropholler, Mislin, Dunwoody und Abels zeigen wir

Satz [L-Meintrup 99] Sei $\overline{G}$ die Komponentengruppe G/G_0, und $\overline{G}_d$ ihre Diskretisierung. Sei $\mathcal{C}om$ bzw $\overline{\mathcal{C}om}$ die Familie der kompakten Untergruppen von $\overline{G}$ und $\overline{\mathcal{C}om}_d$ dieselbe Menge wie $\overline{\mathcal{C}om}$, nun im Zusammenhang mit $\overline{G}_d$. Es gilt

$$\dim(E(G,\mathcal{C}om)) \leq d \Leftrightarrow \dim E(\overline{G},\overline{\mathcal{C}om}) \leq d \Leftrightarrow$$
$$\dim E(\overline{G}_d,\overline{\mathcal{C}om}_d) \leq d \Leftrightarrow \mathrm{hdim}_{\mathbb{Z}\mathrm{Or}(\overline{G}_d,\overline{\mathcal{C}om}_d)} \underline{\mathbb{Z}}$$

wobei $\mathrm{hdim}_{\mathbb{Z}\mathrm{Or}(\overline{G}_d,\overline{\mathcal{C}om}_d)} \underline{\mathbb{Z}}$ die homologische Dimension des konstanten Moduls $\underline{\mathbb{Z}}$ über der Orbitkategorie $\mathrm{Or}(\overline{G}_d,\overline{\mathcal{C}om}_d)$ ist

Satz [L 98] G virtuell torsionsfrei diskret. Sei $l(G) = \sup\{n \mid |H| \mid H \subset G \text{ endlich}\}$
Dann gilt:
$$\mathrm{vcd}(G) \leq d \Rightarrow \dim E(G,\mathcal{F}in) \leq d + l(G)$$

Vermutung [Brown 79] G virtuell torsionsfrei: $\mathrm{vcd}(G) \leq d \Leftrightarrow \dim E(G,\mathcal{F}in) \leq d$

Wolfgang Lück

23 April 1999

Quasi-symmetric functions and the Leibniz-Hopf algebra

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Let $\mathcal{Z}$ denote the Leibniz-Hopf algebra, which is also the Solomon descent algebra and the algebra of noncommutative symmetric functions.

As an algebra $\mathcal{Z} = \mathbb{Z}\langle Z_1, Z_2, \dots \rangle$, the free associative algebra over the integers in countably many indeterminates.

The coalgebra structure is given by

$$\mu(Z_n) = \sum_{i+j=n} Z_i \otimes Z_j \quad , \quad Z_0 = 1$$

Let $\mathcal{M}$ be the graded dual of $\mathcal{Z}$. This is the algebra of quasisymmetric functions ($f(X)$ is quasisymmetric (w.r.t. to the ordering of indeterminates $X_1 < X_2 < \dots$ if for all $j_1 < \dots < j_n$, $k_1 < k_2 < \dots < k_n$ the and exponent sequence $i_1, i_2, \dots, i_n$ the coefficient in f of $X_{j_1}^{i_1} X_{j_2}^{i_2} \cdots X_{j_n}^{i_n}$ is the same as that of $X_{k_1}^{i_1} X_{k_2}^{i_2} \cdots X_{k_n}^{i_n}$. For example $\cancel{X_1X_2}$ $X_1X_2^2 + X_1X_3^2 + X_2X_3^2$ is ~~ny~~ quasi-symmetric but not symmetric in X_1, X_2, X_3).

The Ditters conjecture (1972) states that $\mathcal{M}$ is a free commutative algebra over $\mathbb{Z}$.

In this talk I discuss the Ditters conjecture and give an outline of the recent proof.

Reference:

M Hazewinkel, The algebra of quasi-symmetric functions is free over the integers, preprint March 1999.

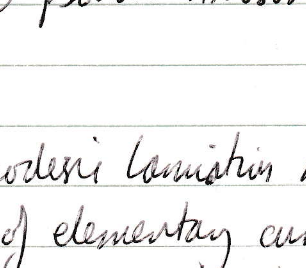
Mapping Class Groups of Punctured Tori

John R. Parker (Durham)

The mapping classgroup (orientation preserving) $MCG(\Sigma)$ of a surface Σ of finite type is the group of all (orientation preserving) homeomorphisms of Σ to itself up to isotopy. A celebrated theorem of Dehn states that $MCG(\Sigma)$ is finitely generated by certain elementary automorphisms, called Dehn twists along certain simple closed curves on Σ. Thurston showed that $MCG(\Sigma)$ acts faithfully on $PML(\Sigma)$, projective measured lamination space, (the so-called Thurston boundary of Teichmüller space). Birman and Series showed how to give $PML(\Sigma)$ a piecewise linear structure (using π_1-train tracks) so that $MCG(\Sigma)$ acts piecewise linearly on $PML(\Sigma)$. Our aim is to describe this explicitly for the once and twice punctured tori. Applications include giving $MCG(\Sigma)$ a concrete automatic structure (known to exist by work of Mosher) or producing an algorithm to decide whether an element of $MCG(\Sigma)$ is (a) periodic, (b) reducible or (c) pseudo-Anosov (following ideas of Hamidi-Tehrani, Chen).

For the once punctured torus, one obtains that any geodesic lamination may be written as a projective linear combination of a pair of elementary curves, written symbolically as $\square\,\square$, $\square\,\square$, $\square\,\square$, $\square\,\square$ corresponding to intervals $[-\infty,-1]$, $[-1,0]$, $[0,1]$, $[1,\infty]$ of $\hat{\mathbb{R}} = \mathbb{R}\cup\{\infty\} = PML(\Sigma_{1,1})$. $MCG(1,1)$ is generated by Dehn twists δ_0, δ_1 about $\square$, $\square$ and has presentation $\langle \delta_0, \delta_1 \mid \delta_0\delta_1\delta_0 = \delta_1\delta_0\delta_1, (\delta_0\delta_1)^3 = e\rangle$. Putting $\delta_0 = \begin{pmatrix}1&1\\0&1\end{pmatrix}$, $\delta_1 = \begin{pmatrix}1&0\\-1&1\end{pmatrix}$ we obtain the well known result that $MCG(\Sigma_{1,1}) = PSL(2,\mathbb{Z})$

For the twice punctured torus, this may be generalised to decompose the 3-sphere $(PML(\Sigma_{1,2}))$ into 28 tetrahedra obtained by putting $\square$ at the north pole, $\square$ at the south pole and triangulating the equatorial plane as indicated



$MCG(\Sigma_{1,2})$ is generated by Dehn twists $\delta_0, \delta_1, \delta_2$ about $\square$, $\square$, $\square$ and has the following presentation

$$\langle \delta_0, \delta_1, \delta_2 \mid \delta_0\delta_1\delta_0 = \delta_1\delta_0\delta_1,\ \delta_0\delta_2\delta_0 = \delta_2\delta_0\delta_2,\ \delta_1\delta_2 = \delta_2\delta_1,\ (\delta_0\delta_1\delta_2)^4 = e\rangle$$

J R Parker 30 April 1999

7.5.99:

A. Suslin, Northwestern:

Polynomial functors and cohomology of algebraic and finite groups.

Finite homotopy types and computation

Graham Ellis (NUI Galway)

This talk describes some computer calculations of 3-dimensional presentations of finite groups of low order. (Such a presentation is equivalent to the 3-skeleton of an Eilenberg-Mac Lane space $X = K(G,1)$.) The method involves applying the LLL algorithm (for finding bases of integer lattices) to the cellular chain complex of the universal cover of X.

This naive approach does not work well if G is large. However, many large G arise from group-theoretic constructions applied to smaller groups. We describe some results on 3-dimensional presentations of such constructions. In particular, we give a result on the 3-presentation of a semi-direct product $G \cong N \rtimes H$.

Our interest in 3-presentations arises from a desire to work towards an explicit enumeration of homotopy types X of "low order". We define the "order" of X to be

$$|X| = \prod_{i \geq 1}^{\infty} |\pi_i X| .$$

If $|X| < \infty$ then clearly $\pi_i X = 0$ for all $i \geq k+1$, some k. Such an X is said to be a "homotopy k-type". Letting

$N(k,m)$ = number of homotopy k-types of order m,

the formula

$$N(k, p^n) = p^{\left\{\frac{(k+1)^{k+1}}{(k+2)!\,(k+2)^{k+2}} n^{k+2} + O(n^{k+1})\right\}}$$

will be explained in the talk.

G. Ellis

14.5.97

The distribution of algebraic numbers

21.05.99

The set of rational numbers is dense in $\mathbb{R}$, but the neighborhood of the rational number α with a small denominator does not contain other rational numbers with small denominators. Therefore any sequence of rational numbers with natural ordering can't be „good" uniform distributed in the sense of good upper bound for the . It is natural to define the new notion of distribution of algebraic numbers. This definition was given by A. Baker and W. Schmidt in 1970. They called it the regular system (RS) and proved that the set of algebraic numbers forms RS: for any interval $I=[a,b]\subset\mathbb{R}$ there exists $K_0=K_0(I)$ such that for every $K>K_0$ we can find real algebraic numbers $\alpha_1, \alpha_2, \dots, \alpha_t \in I$ such that

$$H^{n+1}(\alpha_j)\log^{-\gamma} H(\alpha_j) < K \quad, \quad 1\le j\le t\,, \quad \deg\alpha_j\le n,$$

($H(\alpha_j)$ is the heigt of the algebraic number α_j), with $\gamma = 3n(n+1)$ and the properties

a) $|\alpha_i-\alpha_j| > K^{-1}$, $i\neq j$; b) $t > c(n)(b-a)K$

In 90-th this result was improved ($\gamma=2$) and recently V. Beresnevich obtained $\gamma=0$. These results have a lot of applications in diophantine approximation. In 1998 D. Vasilyev + V.B. constructed best possible RS of algebraic complex numbers. Both results allow to prove a full analogue of the classical theorem of Khintchine about exact order of approximation of almost all real numbers by the rational numbers.

V. Bernik, Institute of Mathematics of Academy of Sciences (Minsk).

28.05.99

Brauer groups and u-invariant of some fields.

The u-invariant of a field F is defined as

$$u(F) = \sup\{\dim q \mid q \text{ is an anisotropic quadratic form over } F \text{ such that } [q] \in \operatorname{Ker}\varphi\},$$

(here φ is a natural homomorphism ~~...~~:

$$\varphi : W(F) \longrightarrow \prod_{v \in \Omega} W(F_v),$$

where Ω is the set of all orderings of F, F_v a real closure of F with respect to $v \in F$, $W(F)$ (resp. $W(F)$) Witt group of F (resp. $W(F_v)$) and $[q]$ the element of $W(F)$ corresponding to q).

In connection with one of Lang's conjectures related to non-formally real function fields (let F be a non-formally real field of transcendence degree i over real closed field, then it is a C_i-field) A. Pfister conjectured in 1982 that if one has a field F of transcendence degree at most 2, over real closed, then $u(F) \le 4$.

The local variant of this problem for function fields is as follows.

Conjecture Let F be a function field of smooth projective curve C defined over ~~the~~ field $R((t))$ of formal power series in t over real closed field R. Then $u(F) \le 4$.

Remark. Note that in case F is non-real it is a local variant of conjecture of Lang for $i = 2$.

We discuss ~~ed~~ some aspect ~~s~~ of positive solution of the conjecture above by using computations in Brauer groups of hyperelliptic curves defined over $R((t))$.

V. Yanchevskii Inst. Math. Acad. Sci Belarus, Minsk

4.6.99

Finiteness properties of arithmetic groups or: reduction, filtration, retraction

Helmut Behr (Frankfurt a. M)

1) Development of reduction theory
   - of quadratic forms
   - for arithmetic groups
   - with compactification (Siegel '59, Borel-Serre '73)
   - with "canonical" filtration (Quillen, Stuhler, Grayson, '72-84) and restraction to the boundary of the unstable region.

2) (Higher) finiteness properties of groups: FP_n and F_n

   a) All S-arithmetic subgroups of reductive algebraic groups over a number field are of type $F(P)_\infty$ (Borel-Serre '76)

   b) Over function fields there exist many counter-examples ~~but~~ and also series of groups with type F_n, not FP_{n+1} (results of Stuhler, Abels, Abramenko, Behr u.a.)

   c) Let G be an alg. group, defined over F, $[F : \mathbb{F}_q(t)] < \infty$, almost simple of rank r over F, S a finite set of primes of F, r_v the rank of G over the completion F_v of F for $v \in S$, Γ a S-arithmetic subgroup of G.

   Question: Is Γ of type F_{n-1}, not FP_n $\Leftrightarrow$ $r > 0$, $\sum_{v \in S} r_v = n$.

   For all results in b, this is true – for some under the assumption, that q is big enough with respect to n.

   d) Program for a proof: Let X be the Bruhat-Tits-building for (G, v), X_∞ the building at infinity, X' the "unstable region" and define a "space of half-lines"

   $Z := \{(x, x') \mid x \in X'_\sigma, x' \in \sigma', \sigma' \; opp_r \; \sigma\}$ by the "opposition-relation" for X_∞, Z has a covering, whose nerve is $Opp_r X$, which ~~has the~~ is r-spherical as X_∞; retract to the (inner) boundary ∂Z: $\partial Z / \Gamma$ finite

   Prop: G almost simple Chevalley gr. over F of rank r, then $\Gamma = G(\mathbb{F}_q[t])$ is of type F_{r-1}, not FP_r.

   Conj: This is also true for Γ S-arithmetic with $\# S = 1$.

H. Behr

11. 06. 99

Isotropy of quadratic forms over function fields of quadrics

Oleg Izhboldin (St. Petersburg / Bielefeld)

Let F be a field of characteristic $\neq 2$. Let φ and ψ be (nondegenerated) quadratic forms over F. We define the order relation $\varphi \precsim \psi$ as follows:

<u>Def</u> 1) $\varphi \precsim \psi$ iff for any field extension L/F ~~we~~ we have: if φ is isotropic over L then ψ is isotropic over L

2) $\varphi \approx \psi$ iff for any field extension L/F we have: φ is isotropic over $L \Longleftrightarrow \psi$ is isotropic over L.

<u>Remark</u> 1) $\varphi \precsim \psi$ <u>if and only if</u> the form ψ is isotropic over the function field of φ. <u>if and only if</u> there exists a rational morphism of quadric X_φ to the quadric X_ψ

2) $\varphi \approx \psi$ if and only if X_φ and X_ψ are stably birational isomorphic.

<u>Examples:</u>
0) if ψ is isotropic then $\varphi \precsim \psi$
1) if φ is similar to a subform of ψ then $\varphi \precsim \psi$
2) if φ is a Pfister neighbor of a Pfister form ψ then $\varphi \approx \psi$
3) if $\varphi = \langle\langle a_1, \dots, a_n\rangle\rangle$ and ψ is a subform of $\tilde{\varphi} = \langle\langle a_1, \dots, a_n\rangle\rangle \otimes \mu$ of ~~codimension~~ dimension $> \frac{1}{2} \dim \tilde{\varphi}$ then $\varphi \precsim \psi$
4) if $\varphi = \langle\langle a_1, \dots, a_n\rangle\rangle \otimes \mu$ ($\dim \mu \geq 2$) and ψ is a subform of φ of codimension $< 2^n$ then $\varphi \approx \psi$

<u>Def</u> Let φ and ψ be such that $\varphi \precsim \psi$. We say that the relation $\varphi \precsim \psi$ is <u>standard</u> if there exist $\varphi_0, \varphi_1, \dots, \varphi_k$ s.t. $\varphi = \varphi_0 \precsim \varphi_1 \precsim \dots \precsim \varphi_k = \psi$ and $\forall\ i = 1, \dots, k$, the relosion $\varphi_{i-1} \precsim \varphi_i$ belonds to list of <u>Examples 0–4</u>.

<u>Theorem 1</u> Let φ and ψ be such that $\varphi \precsim \psi$. Then "$\varphi \precsim \psi$ is standard" in the following cases
1) ψ is a Pfister neighbor
2) $\dim \psi \leq 5$
3) $\dim \psi = 6$, except for the case where $\begin{cases} \dim \varphi = 4 \text{ and} \\ 1 \neq \det \varphi \neq \det \psi \neq 1 \\ \operatorname{ind} C_0(\varphi) = \operatorname{ind} C_0(\psi) = \operatorname{ind} C_0(\varphi) \otimes C_0(\psi) = 4 \end{cases}$
4) $\dim \psi = 8$, $\psi \in I^2 F$, $\dim \varphi \geq 5$
5) $\dim \psi \leq 2^n$ and $\dim \varphi > 2^n$
6) $\dim \psi = 2^{n+1}$ and $\dim \varphi \geq 2^n - 7$.

<u>Theorem 2</u> there exists a "nonstandard relation $\varphi \precsim \psi$" with $\dim \varphi = 6$.

The proofs of Theorems 1 and 2 uses methods of algebraic K-theory, Galois cohomology an CH^*-groups of quadrics (and products of quadrics)

Олег Ижболдин

18.06.99

An excursion into analysis and irregularity

Umberto Mosco (Roma)

Analysis has been confronted with "irregularities" of various kind all along this century.
"Irregularity" is used here in a broad sense, referring to analytic behavior deviating in a substantial way from classic Euclidean or Riemannian models.
A good insight into a large class of non-Euclidean structures can be obtained by focussing on basic concepts like length, volume energy and on the mutual relations existing among them. The analytic tools for such a general theory can be found in the fundamental works of De Giorgi and Moser on uniformly elliptic operators in divergence form and measurable coefficients, of John – Nirenberg on BMO spaces of functions with bounded mean oscillation and in Hörmander's ipoellipticity theory for smooth vector fields and related stratified Lie groups, like the Heisenberg group and in further developments from a metric point of view by Rothschild – Stein, Fefferman – Phong, Nagel – Stein – Wainger, Jerison and others, in the last two decades.
The conclusion we can draw by analysing these theories closely, as well as the recent mathematical and physical work on fractals, can be summarized as follows:
Various classical, semiclassical and fractal theories of "elastic bodies" possibly of very irregular nature can be built on a common metric background,

involving locally compact topological spaces X and

- quasi-distances
$$d(x,y) \le c_T\,[d(x,z) + d(z,y)] \quad , \quad c_T \ge 1$$
- doubling measures
$$\mu(B_r) \ge c_1\, \mu(B_R)\left(\frac{r}{R}\right)^{\nu} \quad , \quad \nu > 0 \ ,$$
$B_r = B_r^d$, $0 < r \le R$
- measure-valued gradient forms ("Lagrangians")
$$\mathcal{L}(u,v) = \text{"}\nabla u \cdot \nabla v\text{"}$$

defined on dense subalgebras $\mathcal{C}$ of $C_0(X)$, all of them related mutually by the property:

Lagrangians control bounded-mean-oscillations at all metric scales;

$$⨍_B |\varphi - \bar{\varphi}_B|\, d\mu \le c_2\, R \left(⨍_{qB} d\mathcal{L}(u,u)\right)^{1/2} \quad , \quad u \in \mathcal{C}$$

$$⨍_B = \mu(B)^{-1}\int_B \quad , \quad \bar{\varphi}_B = ⨍_B \varphi\, d\mu \quad , \quad B = B_R \quad , \quad q \ge 1 \, .$$

All previous properties taken together give to $X, d, \mu, \mathcal{L}$ the nature of a "pseudo-Riemannian" space. The main analytic procedures leading to these structures are Lie-group invariance in the smooth case and self-similarity invariance in the fractal case.

Vielen Dank!, U.M.

25.06.99

Complex geometry of real symmetric spaces.
Simon Gindikin (Rutgers University & MPI)

Let $X_0 = G_0/K_0$ be a ~~real~~ Riemann symmetric space, $X = G/K$ be its complexification. There is a canonical Stein manifold $X_0 \subset Y \subset X$, satisfying to the condition: all zonal spherical functions on X_0 admit holomorphic extensions on Y and it is a maximal manifold with such a condition. We call Y by the crown of X_0: $Y = \mathrm{Crown}(X_0)$. Many other objects admit holomorphic extensions on Y: eigen functions of Laplace operators, solutions of Schmid equations (in which discrete series of representations can be realized), kernels of Szegö operators etc.

There are several geometric conjectures which are checked only at some examples. Crowns are G_0-invariant, but not homogeneous. There is a hypotetick conjecture on the paramatrization of G_0-orbits. There is a conjecture that Y gives a parametrization of complex cycles at flag domains. It's proved for groups of Hermitian type and for $SL(n, \mathbb{R})$, $SU(1,n)$ and a few examples.

The second subject of the tack – tube domains which edges are pseudo Riemannian symmetric spaces. The conjecture is that it is possible to define Hardy spaces of $\bar\partial$-cohomology in which different series of representations can be realized. Between these tubes some are Stein manifolds which correspond either holomorphic discrete series or maximal continuous series on causal symmetric spaces. In the ~~case~~ last case tubes coincide with crowns of corresponding Riemannian symmetric spaces.

Simon Gindikin

25·06·99

Restriction Maps between Cohomologies of Arithmetic Groups.

T. N. Venkataramana
(School of Math., Tata institute of Fundamental Research, Homi Bhabha Road, Colaba, MUMBAI 400 005, INDIA).

We establish a criterion to determine when a cohomology class $\omega \in H^\cdot(S(\Gamma), \mathbb{C})$, of a smooth compact locally Hermitian symmetric variety $S(\Gamma)$ (Γ is a torsion-free arithmetic subgroup of the group G of holomorphic automorphisms of a Hermitian symmetric domain of noncompact type), "restricts" non-trivially to a smaller complex submanifold $S_H(\Gamma)$ of $S(\Gamma)$ (here $S_H(\Gamma)$ is also a compact locally symmetric variety). Here, the "restriction" is in the sense that one may have to move the class ω by a Hecke correspondence and then to restrict the resulting class to $S_H(\Gamma)$.

Let $\hat{X}$ and $\hat{Y}$ be the compact dual symmetric spaces of $S(\Gamma)$ and $S_H(\Gamma)$, respectively; let $[\hat{Y}]$ be the Poincaré dual of $\hat{Y}$ in $H^\cdot(\hat{X})$. Via the Matsushima Formula, $H^\cdot(\hat{X})$ may be thought of as a subring of $H^\cdot(S(\Gamma))$. Let $[S_H(\Gamma)]$ denote the Poincaré dual of the special cycle $S_H(\Gamma)$ in $S(\Gamma)$.

__Theorem 1:__ 1) The class $[\hat{Y}]$ is a linear combination of Hecke translates of $[S_H(\Gamma)]$.

2) If all the Hecke translates of a class ω restrict trivially to $S_H(\Gamma)$, then $\omega \wedge [\hat{Y}] = 0$.

Using Theorem 1, we may show

__Theorem 2:__ If X (resp. Y) is the unit ball in $\mathbb{C}^n$ (resp. the unit ball in $\mathbb{C}^m$), then all the cohomology of $S(\Gamma)$ upto degrees $\leq \dim Y$, "restrict" injectively to $S_H(\Gamma)$.

This affirms a conjecture of M. Harris and J.-S. Li.

Theorem 1 also implies the following:

Theorem 3: Let $S(\Gamma)$ be a quotient of the unit ball in $\mathbb{C}^n$, with non-trivial first Betti number, and let $Z \subset S(\Gamma)$ be a (closed) smooth subvariety. Then there exists a finite covering $\tilde{Z}$ of Z such that all its Hodge numbers $h^{p,q} \doteqdot \dim_{\mathbb{C}} H^{p,q}(\tilde{Z})$ are nonzero (for all $p, q \leq \dim Z$).

To prove Theorem 3, one notes that restricting a tensor product $\omega \otimes \omega' \in H^{\cdot}(S(\Gamma) \times S(\Gamma))$ (of classes ω and ω' on $H^{\cdot}(S(\Gamma))$) to the diagonal $S(\Gamma)$, is merely taking the cup-product $\omega \wedge \omega'$ of these classes. Theorem 3 is then shown to follow by using the criterion of Theorem 1.

[illegible]

6.07.99

Hardy theorem for some Lie groups
— a version of uncertainty principle —

Keisaku KUMAHARA (The Univ. of the Air, Chiba)

The uncertainty principle says that a function is concentrated, th its Fourier transform cannot be concentrated unless it is identicall zero. The most well-known theorem is the Heisenberg - Pauli - Weyl inequality. And many generalizations and variations a known. One of them is the Hardy theorem, which yields that if a measurable function f on $\mathbb{R}$ satisfies

$$|f(x)| \leq C \exp(-ax^2), \quad |\hat{f}(\xi)| \leq C\exp(-b\xi^2)$$

for $C>0$, $a>0$, $b>0$ and $ab>\frac{1}{4}$, then $f=0$ (a.e.). Here we take $\hat{f}(\xi) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} dx$ as the definition of the Fourier transform. M.G. Cowling and J.F. Price an L^p version of the Hardy theorem (1983): If a measurable function f on $\mathbb{R}$ satisfies $\| e^{ax^2} f \|_p < \infty$, $\| e^{b\xi^2} \hat{f} \|_q < \infty$ for some p, q which are $1 \leq p, q \leq \infty$ ~~and~~, one of them are finite, and $ab \geq \frac{1}{4}$, then $f=0$ (a.e.)

In this talk I report some generalizations of such theorem.

(1.) an analogue of the Hardy theorem for Cartan motion group

(2.) an analogue of the Hardy theorem for connected noncomp semisimple Lie groups

(3.) an L^p-version of the Hardy theorem for the motion group

(1) ~~is go~~ and (3) are joint works with M. Eguchi and S. Koizumi. (2) is joint work with M. Ebata, M. Eguchi and S. Koizumi.

Keisaku Kumahara

熊原啓作

15. 10. 99

CURVATURE NOTIONS IN GROUPTHEORY

(JENS HARLANDER, UNIV. FRANKFURT)

METRIC AND GEOMETRIC STRUCTURES ON CELL COMPLEXES HAVE A LONG HISTORY, POSSIBLY BEGINNING WITH DEHN'S ALGORITHM FOR SOLVING THE WORD PROBLEM IN SURFACE GROUPS. IN GENERAL, GIVEN A CW-COMPLEX, ONE CAN METRIZE THE INDIVIDUAL CELLS AND THEN EXTEND THE LOCAL METRICS ON THE WHOLE COMPLEX. CURVATURE CONSIDERATIONS FOR METRIC SPACES APPEAR FIRST IN THE FUNDAMENTAL WORK OF ALEXANDROV. THESE NOTIONS WERE RE-INTRODUCED TO MAIN STREAM GEOMETRIC GROUP THEORY BY GROMOV IN CONNECTION WITH HYPERBOLIC GROUPS. IN MANY CASES, COMPLICATED HOMOTOPICAL OR COMBINATORIAL PROOFS THAT ESTABLISH ASPHERICITY CAN BE CONSIDERABLY SIMPLIFIED USING GEOMETRIC TECHNIQUES.

IN MY TALK I WILL DISCUSS METRIC AND NON-METRIC ASPHERICITY TESTS BASED ON CURVATURE NOTIONS.
I WILL ILLUSTRATE THE TECHNIQUES BY GIVING APPLICATIONS TO ARTIN AND COXETERGROUPS, KNOT AND RIBBON-DISC COMPLEMENTS.

Jens Harlander

22.10.93

Nichtkommutative Charaktere: ein neuer Zugang zur Charaktertheorie der symmetrischen Gruppen.

Die Charaktertheorie der symmetrischen Gruppen läßt sich elegant und übersichtlich organisieren mit Hilfe der Bialgebra $\mathcal{C}$ der Klassenfunktionen, wie Geissinger 1977 gezeigt hat. Im einzelnen bedeutet dies: Wir sehen $\mathcal{C} := \bigoplus \mathcal{C}\ell_K(S_n)$ — die direkte Summe aller Vektorräume der K-wertigen Klassenfunktionen auf den symmetrischen Gruppen S_n. $\mathcal{C}$ ist bei stellweiser Multiplikation eine Algebra $(\mathcal{C}, *)$. Tensorprodukt und Induktion liefern eine weitere Multiplikation $\bullet$ auf $\mathcal{C}$. Ein Coprodukt $\downarrow : \mathcal{C} \to \mathcal{C} \otimes \mathcal{C}$ ergibt sich durch Reduktion der Klassenfunktionen auf geeignete Young-Untergruppen (parabolische Untergruppen). Die üblichen Bilinearformen auf den Räumen $\mathcal{C}\ell_K(S_n)$ setzt man zu einer Bilinearform auf $\mathcal{C}$ zusammen. Die klassische Charaktertheorie ist dann die Theorie des Rechnens in $(\mathcal{C}, *, \bullet, \downarrow, (\cdot,\cdot)_{\mathcal{C}})$, wobei das Rechnen mit irreduziblen Charakteren das Hauptproblem ist.

Ein neuer Zugang wird durch die Konstruktion einer geeigneten Bialgebra mit Skalarprodukt $(\mathcal{R}, \circ, \downarrow, (\cdot,\cdot)_{\mathcal{R}})$ und eines Algebrenepimorphismus $c : (\mathcal{R}, \circ) \to (\mathcal{C}, \bullet)$ gegeben, für den außerdem $c(\varphi)\downarrow = (c \otimes c)(\varphi\downarrow)$ und $(\varphi,\psi)_{\mathcal{R}} = (c(\varphi), c(\psi))_{\mathcal{C}}$ gilt. $\mathcal{R}$ wird erzeugt von den Summen $Z^{p\wedge q}$ aller Standard-Young-Tableaux der jeweiligen Gestalt $p\wedge q$, wobei p, q Partitionen sind. Setzt man $Z^p := \sum t$, $t \in SYT^p$, wobei SYT^p die Menge aller Standard-Young-Tableaux der Gestalt p, p eine Partition von n ist, so ist $\zeta^p := c(Z^p)$ derjenige irreduzible Charakter von S_n, dessen Bild unter dem Frobenius-Isomorphismus gerade die Schurfunktion s_p ist. Der Satz von Littlewood-Richardson erscheint jetzt in der Gestalt

$$(\zeta^p \bullet \zeta^q, \zeta^r)_{\mathcal{C}} = (Z^p \circ Z^q, Z^r)_{\mathcal{R}},$$

eine Form dieses Satzes die 1985 von Garsia und Remmel angegeben wurde. Auch die anderen Sätze der klassischen Charaktertheorie lassen sich mit Hilfe dieses nichtkommutativen Überbaus von $\mathcal{C}$ leicht herleiten. Solomons descent algebra ist enthalten in $\mathcal{R}$ und c eine Fortsetzung von Solomons Epimorphismus. Dieser neue Zugang entstammt der Kieler Dissertation von A. Jöllenbeck von 1998 und ist mittlerweile veröffentlicht in den Bayreuther Mathematischen Schriften unter dem Titel: Nichtkommutative Charaktertheorie der symmetrischen Gruppen.

Dieter Blessenohl, Universität Kiel

29/10/99

Dynamical Systems and Controllability: A Systems Theory View

This talk proposes some mathematical concepts for 'open' dynamical systems. We start with a specific example, the problem of obtaining a mathematical model for a simple RLC-circuit. This leads to the problem of how to capture the resulting 'first principles' model into mathematics. After ~~a~~ brief historical remarks, we end up with a definition in terms of a behavior defined by equations that involve both manifest and latent variables. We subsequently restrict attention to linear shift-invariant differential systems.

Three specific problems are discussed: 1. the elimination problem 2. controllability, and 3. observability. The elimination theorem basically states that the behaviors of linear shift-invariant differential systems are closed under (intersection, addition, and) projection. The next topic addressed is controllability, which is defined as a 'patching' property. We show that controllability is equivalent to the existence of an image representation, i.e., of a potential function. We finally discuss, very briefly, the 'dual' notion of observability.

As closing remarks, we allude to some applications of these ideas in control and filtering, coding (convolutional codes), to first order representations, and to the theory of dissipative structures.

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5.11.1999

Zur globalen Dynamik ~~ebener~~ polynomialer Vektorfelder

Die Untersuchung der globalen Dynamik ebener (und erst recht höherdimensionaler) polynomialer Vektorfelder entbehrt bisher einer festen und einheitlichen Grundlage — wie etwa die algebraische Geometrie ~~bzw~~ kommutative Algebra "erfunden" wurde.

Im Vortrag soll eine neuartige Methode der "Pseudoorbits im Pseudophasenraum" bzw. der "Pseudopotentiale" dargestellt werden, welche sich am Bsp. einer gewissen Klasse ebener polynomialer Vektorfelder bewährt hat.

Für solche ebenen Vektorfelder nämlich, welche sich durch die Reellifizierung komplex-eindimensionaler ~~Vektor~~ polynomialer Vektorfelder aller Grade ergeben, liefert die neue Methode u.a. folgende Ergebnisse:

1) Lokale und globale Fragen können auf einer gemeinsamen Grundlage behandelt werden.
2) Nicht-existenz von Grenzzyklen,
3) Vollständige kombinatorische Beschreibung aller möglichen topologischen Typen von Flüssen,
4) Nachweis, dass alle kombinatorischen Typen auch tatsächlich durch Flüsse realisierbar sind, d.h. vollständige topologische Klassifikation.
5) Eine neue Familie globaler Bifurkationen tritt auf.

etc. etc.

Ein neuartige Form von point plots für die Lösungskoeffizienten der lokalen Potenzreihenlösungen soll dargestellt werden, mit der man erkennt, dass eine wichtige Voraussetzung für die Anwendbarkeit der Pseudo orbit-methode universell für alle polynomialen Grade und Dimensionen erfüllt ist. Diese point plots spielen in Funktion (jedoch nicht in ihrer Form) die gleiche ~~Rolle~~ wie Julia mengen in der Iteration polynomialer Abbildungen.

Dr. Rudolf Winkel

RWTH-Aachen

12.11.99

Generische Initialideale und graduierte Bettizahlen

In diesem Vortrag werden Sätze von G. Kalai über "algebraic shifting" verallgemeinert. Zu einem gegebenen simplizialen Komplex Δ definiert Kalai einen "geshifteten" Komplex Δ^c wie folgt: Man ~~betrachtet~~ betrachtet in der äußeren Algebra das Stanley Reisner Ideal $J_\Delta \subset E$

Dann ist Δ^e (der geshiftete Komplex) definiert durch die Gleichung $J_{\Delta^e} = \mathrm{Gin}(J_\Delta)$. Hierbei bedeutet Gin das generische Initialideal. Bei dem Übergang von Δ zu Δ^e bleiben viele Eigenschaften von Δ erhalten, etwa die simpliziale Homologie oder auch die Cohen-Macaulay Eigenschaft. Δ^e ist aber kombinatorisch einfacher zu behandeln. Mit der Methode des "algebraic shifting" haben Björner und Kalai ein verallgemeinertes Euler-Poincaré Theorem bewiesen. In diesem Vortrag soll ein gemeinsames Resultat mit Aramova vorgestellt werden, das besagt, daß Δ und Δ^e dieselben extremalen Bettizahlen besitzen. Dieses Resultat ist eine weitgehende Verallgemeinerung der Sätze von Kalai. Zum Abschluß werden einige Anwendungen gegeben.

Jürgen Herzog

Univ. Essen

19/11/1999

"B_{n-1}-orbits in GL_n/B_n"

T. Hashimoto

Let k be a field of characteristic zero, and $GL_n := GL_n(k)$. We embed GL_{n-1} into GL_n by the map $t \mapsto \begin{pmatrix} t & \\ & 1 \end{pmatrix}$. Then denoting by B_n the Borel subgroup of all upper triangular matrices in GL_n, B_{n-1} acts on the flag variety GL_n/B_n with an open dense orbit. In this talk I give an algorithm to determine all the B_{n-1}-orbits, and describe the "Bruhat order", i.e. the closure relation between the orbits in a fixed Bruhat cell. As a corollary, one finds ~~an~~ the orbits whose closures coinsides with the Schubert varieties.

橋本隆司
Takashi Hashimoto
(Tottori Univ. / Strasbourg Univ.)

3 december 1999

Units in Integral Group Rings

The integral group ring $\mathbb{Z}G$ of a group G is a natural object where group- and ring theory meet. Quite recently M. Hertweck gave a counterexample to the old isomorphism problem, that is, he constructed two non-isomorphic finite groups G and H so that $\mathbb{Z}G$ and $\mathbb{Z}H$ are isomorphic. This again gave a boost to the study of the unit group of an integral group ring.
Very recently, Mazur, Juriaans and Jespers showed that there is a close relationship between the isomorphism problem and the way the group G is embedded in $U(\mathbb{Z}G)$, the units. In particular, the isomorphism problem holds for $G \times A$, with A a finitely generated free abelian group (non-trivial) (G a finite group) if and only if the isomorphism problem and normaliser problem holds for G, that is, the normaliser of G in $U(\mathbb{Z}G)$ equals $\pm G\, Z(U(\mathbb{Z}G))$ (the trivial units times the central units). We will survey some of these results.

Also we survey some of the recent results on the big lacking information of $U(\mathbb{Z}G)$, G a finite group: namely, find a structure theorem of the unit group. For most finite groups G we know a finite set of constructive generators of a subgroup of finite index in $U(\mathbb{Z}G)$ but we lack knowledge on the structure. However, one can classify the finite groups G so that $U(\mathbb{Z}G)$ contains a subgroup of finite index that is a free product of abelian groups. The latter is joint work with del Rio and Leal.

Eric Jespers
Vrije Universiteit Brussel
Belgium

Let G be a connected, reductive, linear algebraic group over $\mathbf{C}$ and let σ be an involutorial automorphism of G, with fixed point group H. The quotient G/H is a *symmetric variety.*
A Borel group B of G has finitely many orbits on G/H (i.e. the number of double cosets BgH is finite). Let V be the set of these orbits. It has a partial order, induced by inclusion of orbit closures.
In joint work with R.W. Richardson (Geom. Dedic. 35 (1990), 389-436) a combinatorial set-up was introduced, in order to study the ordered set V. It turns out that this set has some resemblance to the Weyl group of G, with its 'Bruhat order'.
The symmetric variety G/H has a 'wonderful' compactification X, introduced by De Concini and Procesi in 1983. It is a smooth, projective, G-variety containing G/H as an open subvariety. B has finitely many orbits on X, let $\bar{V}$ be the ordered set of these orbits.
In the talk I discussed the extension to $\bar{V}$ of the combinatorial set-up for V. The results are not yet as complete as one would wish.

T.A. Springer
Universiteit Utrecht

T A Springer

26. 11. 99

De

[illegible]

Dec. 10, 1999

Codes with a large gap in the weight distribution

The talk is concerned with the probability of an incorrect decoding when an $[n,k,d]_q$ code C is used for error correction. If the channel has symbol error probability p the probability we are interested in is the conditional probability
$F(p,t;C) := P(\text{incorrect decoding} \mid d(\text{received word}, \text{decoded word}) \leq t)$
where $t \leq \frac{d-1}{2}$ and $d(\cdot,\cdot)$ denotes the Hamming distance.
For small p and $[n,k,d]_q$ resp. $[n,k,d']_q$ codes C resp. C' with weight distribution $(A_0,\ldots,A_n)$ resp. $(A_0',\ldots,A_n')$, A. Faldum recently proved:
$F(p,t;C) < F(p,t;C')$ if and only if
$(A_0,\ldots,A_n) < (A_0',\ldots,A_n')$ in lexicographical order. Obviously, MDS codes have minimal decoding error probability in the class of all $[n,k]_q$ code.
We present a class of codes, so-called MMD codes, i.e. codes with weight distribution $(A_0,\ldots,A_d,0,0,\ldots 0,A_{n-d^\perp+2},\ldots A_n)$ which have minimal decoding error probability in the class of all $[n,k,d]_q$ codes. (Here $d^\perp$ denotes the minimum distance of the dual code). The classification of such MMD codes mainly relies on the rigid structure which the set of column vectors of a generator matrix has to satisfy.

Wolfgang Willems
Otto-von-Guericke Universität Magdeburg

17.12.99

Verallgemeinerte Standardtableaux in der Darstellungstheorie endlicher Gruppen

Nach dem Satz von Wedderburn ist die komplexe Gruppenalgebra einer endlichen Gruppe G isomorph zu einer Algebra von Blockdiagonalmatrizen: $D: \mathbb{C}G \to \bigoplus_{k=1}^{h} \mathbb{C}^{d_k \times d_k}$. Jeder derartige Algebrenisomorphismus heißt eine diskrete Fouriertransformation (DFT). Die h Projektionen $D_1, \ldots, D_h$ von D bilden ein vollständiges System paarweise inäquivalenter irreduzibler Darstellungen von $\mathbb{C}G$. In dem Vortrag gehe ich folgenden Fragen nach:

- Wie kann man schnell irreduzible Darstellungen konstruieren?
- Welche DFTs lassen sich schnell auswerten?

Erstaunlicherweise hängen diese beiden Fragen eng zusammen. Verbindendes Element sind kombinatorische Strukturen, die Verallgemeinerungen der aus der Darstellungstheorie symmetrischer Gruppen bekannten Standardtableaux sind. Diese verallgemeinerten Standardtableaux dienen als Organisationskonzept für effiziente Algorithmen sowohl zum Generieren von DFTs als auch zum Auswerten von DFTs.

Schließlich besprechen wir einige Anwendungen:

- schnelles Falten in Gruppenalgebren
- Collection in p-Gruppen mittels DFTs
- Berechnung von Charaktertafeln.

Michael Clausen

(Universität Bonn)

(17.12.99)