Workshop on Funke–Millson Theory

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1 Plan of the workshop

The aim of this workshop is to understand the work of Funke–Millson [2, 4]. A good overview can be found in [1]. Its relation to the Shintani lift can be found in [3], where the Shintani lift is extended to arbitrary modular forms.

	Monday	Tuesday	Wednesday
9:00 - 10:15	Talk 1	Talk 5	Talk 9
10:30 - 11:45	Talk 2	Talk 6	Talk 10
11:45 - 13:30	Lunch	Lunch	Lunch
13:30 - 14:45	Talk 3	Talk 7	Talk 11
14:45 - 16:00	Talk 4	Talk 8	
16:00 - 17:00	Discussion	Discussion	

1. Overview

2. Representations of GL_n and O_n [2, Section 3], [4, Section 3]:

   Introduce Young diagrams and Young tableaus. Define the irreducible representations corresponding to a Young tableau. Define the Schur functors and prove [4, Corollary 3.4]. Discuss the harmonic projections and harmonic Schur functors.

3. Special cycles with local coefficients [2, Section 4]:

   Introduce locally symmetric spaces of orthogonal type, that is, locally symmetric spaces attached to orthogonal groups of quadratic spaces. Define special cycles and discuss neat subgroups. Promote the special cycles to cycles with non-trivial coefficients. Introduce composite cycles.

4. Funke–Millson Schwartz Form [2, Section 5]:

   Introduce the double complex for the Weil representation. Define the Funke–Millson Schwartz form. State the fundamental properties of the Funke–Millson Schwartz form.

5. Fundamental properties of the Funke–Millson Schwartz form I [2, Section 6, Appendix A]:

Introduce the Fock model of the Weil representation and the splitting of the Funke–Millson Schwartz form in the Fock model. Prove that it is K'-invariant [2, Theorem 6.2] and that it is closed [2, Theorem 5.7, Theorem 6.3] assuming the necessary results of [5].

6. Fundamental properties of the Funke–Millson Schwartz form II [2, Section 6]:

   Prove the recursion formula [2, Theorem 5.9, Theorem 6.4] and show that the Funke–Millson Schwartz form is holomorphic [2, Theorem 5.10, Theorem 6.11] assuming the necessary results of [6]

7. Modularity of Special Cycles with coefficients [2, Section 7]:

   Introduce the Funke–Millson theta form. Prove the main Theorems [2, Theorem 7.5, 7.6, 7.7] assuming the necessary results of [6].

8. Borel-Serre Compactification [4, Section 2]:

   Describe the Langlands decomposition of a parabolic subgroup and the horospherical coordinates. Introduce the Borel-Serre compactification of symmetric spaces of orthogonal type. Feel free to ignore the $\mathbb{Q}$-split case.

9. Local Restrictions [4, Section 4.2, Section 5]:

   Introduce the mixed model of the Weil representation and define the local restriction of a Schwartz function. Define the local restriction map to the differential graded algebra $A_p^\bullet$.

10. The map $i_P\colon C_W^\bullet \to A_P^\bullet$ [4, Section 6]:

    Introduce the map i_P and prove [4, Proposition 6.15].

11. Global Restrictions [4, Section 8, Section 9]:

    Calculate the local restrictions of the Funke–Millson Schwartz form. Introduce the global restriction maps and prove the restriction formula [4, Theorem 9.5, Corollary 9.7].

References

[1] Jens Funke. Special cohomology classes for the weil representation. In *Automorphic Representations, Automorphic Forms, L-functions, and Related Topics*, volume 1617, pages 106–119. RIMS Kôkyûroku, 2008.

[2] Jens Funke and John Millson. Cycles with local coefficients for orthogonal groups and vector-valued Siegel modular forms. *Amer. J. Math.*, 128(4):899–948, 2006.

[3] Jens Funke and John Millson. Spectacle cycles with coefficients and modular forms of half-integral weight. In *Arithmetic geometry and automorphic forms*, volume 19 of *Adv. Lect. Math. (ALM)*, pages 91–154. Int. Press, Somerville, MA, 2011.

[4] Jens Funke and John Millson. Boundary behaviour of special cohomology classes arising from the Weil representation. *J. Inst. Math. Jussieu*, 12(3):571–634, 2013.

[5] Stephen S. Kudla and John J. Millson. The theta correspondence and harmonic forms. I. *Math. Ann.*, 274(3):353–378, 1986.

[6] Stephen S. Kudla and John J. Millson. Intersection numbers of cycles on locally symmetric spaces and Fourier coefficients of holomorphic modular forms in several complex variables. *Inst. Hautes Études Sci. Publ. Math.*, (71):121–172, 1990.