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1. Logic

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The enumeration degree structures

A set A is enumeration reducible (e-reducible) to a set B if there exists a recursively enumerable (r. e.) set Φ (called in this context an e-operator) such that $A = \{x : (\exists u)[\langle x, u\rangle \in \Phi \,\&\, D_u \subseteq B]\}$.

The structure $\mathcal{D}_e$ of the e-degrees is the structure of the equivalence classes (called e-degrees) of sets of numbers under the equivalence relation $\equiv_e$ generated by the preordering $\leq_e$; the e-degree of A is denoted by the symbol $\deg_e(A)$. $\mathcal{D}_e$ is in fact an upper semilattice with least element $\mathbf{0}_e$, where $\mathbf{0}_e = deg_e(W)$, any r. e. set W.

We study the structures $\mathcal{D}_{\alpha,e}$ of α-r. e. e-degrees for $\alpha \leq \omega$. In [CSY] it was shown that for any Δ^0_2-e-degree $\mathbf{a} >_e \mathbf{0}_e$ there exists a Δ^0_2-e-degree $\mathbf{b} <_e \mathbf{0}'_e$ such that $\mathbf{0}'_e = \mathbf{a} \cup \mathbf{b}$. In [AS] we showed that here in general we can not choose $\mathbf{b}$ as an ω-r. e. e-degree: there is a Δ^0_2-e-degree $\mathbf{a} >_e \mathbf{0}_e$ such that $\mathbf{a} \cup \mathbf{b} <_e \mathbf{0}'_e$ for any ω-r. e. e-degree $\mathbf{b} <_e \mathbf{0}'_e$.

We will discuss further results of this kind. In particular, we prove that structures of n-r. e., ω-r. e. and total e - degrees for $n < \omega$ are not pairwise elementarily equivalent, and discuss the analog of Downey's Conjecture for the structures of n-r. e. e-degrees, $1 < n < \omega$.

References: [CSY] S. B. Cooper, A. Sorbi, and X. Yi. Cupping and non-cupping in the enumeration degrees of Σ^0_2-sets. To appear.

[AS] M. M. Arslanov, A. Sorbi. A note on the Δ^0_2 enumeration degrees. To appear.