ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

1. Logic

1991 MS Classification:

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Valuations on Hilbert algebras

A *Hilbert algebra* (Ha) is a triple (A, $\rightarrow$,1), where A is a non set, $\rightarrow$ is a binary operation on A, $1 \in$ A is an Element such that the next axioms is verified for every x, y, z $\in$ A :

(1) $x \rightarrow (y \rightarrow x) = 1$

(2) $(x \rightarrow (y \rightarrow z)) \rightarrow ((x \rightarrow y) \rightarrow (x \rightarrow z)) = 1$

(3) If $x \rightarrow y = y \rightarrow x = 1$, then $x = y$

Examples 1. Classical propositional calculus, where $\rightarrow$ denotes the logical implication.

2. If (A, $\leq$) is a posed with 1, then (A, $\rightarrow$,1) is a Ha, where for x, y $\in$ A $x \rightarrow y = 1$ iff $x \leq y$ and $x \rightarrow y = y$ if $x \leq y$.

3. A Heyting algebra is a lattice A with 0 such that for every x, y $\in$ A there exists an element $x \rightarrow y \in A$ with the property that for $z \in A$, $x \wedge z \leq y$ if $z \leq x \rightarrow y$.

If (A, $\vee$, $\wedge$, 0) is a Heyting algebra, then (A, $\rightarrow$, 1) is a Ha, where $1 = a \rightarrow a$ for an $a \in A$.

4. If (A, $\vee$, $\wedge$, $\urcorner$, 0, 1) is a Boolean lattice, then (A, $\rightarrow$, 1) is a Ha, where for x, y $\in$ A, $x \rightarrow y = (\urcorner x) \vee y$.

The origin of Hilbert algebras is in Mathematical Logic .

In 1923 Hilbert was the first who remarked the possibility of study a very interesting part of the classical calculus, taking as axioms only the Hilbert' s ones concerningonly for logical implication (this field is known as "positive implicative propositional calculus") .

If (A, $\rightarrow$, 1) is a Ha, we define *pseudo-valuation* on A as rea-function v on A satisfing $v(1) = 0$ and $v(x \rightarrow y) \geq v(y) - v(x)$ for any x, y $\in$ A (v is called a *valuation* if $x = 1$ whenever $v(x) = 0$) .

the aim of my communication is to prove that every pseudo-valuation (valuation) v includes a pseudo-metrics (metric) d_V on A definite by

$d_V(x, y) = v(x \rightarrow y) + v(y \rightarrow x)$

under wich $\rightarrow$ is uniformly continuos .

Also, is proved many theorms of extension for pseudo-valuation .

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