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Abstracts of Short Communications and Posters

Section:

2. Algebra

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On decomposing some finitely generated groups into amalgamated free products

A decomposition of a group Γ as a non-trivial free product with amalgamation (an FPA decomposition) can often be used to obtain important properties of Γ. Let $X_2^S(\Gamma)$ be the variety of characters of irreducible representations of Γ in $\mathrm{SL}_2(\mathbb{C})$.

Theorem 1. *If* $\dim X_2^S(\Gamma) \geq 2$ *then* Γ *has an FPA decomposition.*

C. T. C. Wall posed the following question (1977): which one-relator groups have FPA decompositions? Baumslag and Shalen [1] proved that if such a group G has ≥ 3 generators then G has an FPA decomposition. If G is an one-relator two generator group with torsion then obviously $\dim X_2^S(G) \geq 2$ and by Theorem 1 G is an FPA group (which proves conjecture of Fine, Levin and Rosenberger [2]).

Theorem 2. *If* $G = \langle a, b \mid a^{2n} = R^m(a,b) = 1\rangle$, *where* $n \geq 0$, $m \geq 2$ *and* $R(a,b)$ *is a cyclically reduced word involving* b *then* G *has an FPA decomposition.*

As a corollary we obtain that Fuchsian groups $G_1 = \langle a, b \mid [a,b]^n = 1\rangle$, $G_2 = \langle a, b \mid a^2 = [a,b]^n = 1\rangle$ have FPA decompositions for any $n \geq 2$.

References: 1. G. Baumslag, P. Shalen, Amalgamated products and finitely presented groups, Comment. Math. Helv. **65**(1990), 243–254. 2. B. Fine, F. Levin, G. Rosenberger, Free subgroups and decompositions of one-relator products of cyclics; Part 2: Normal torsion free subgroups and FPA decompositiomns, J. of Indian Math. Soc. **49**(1985), 237–247.