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Abstracts of Short Communications and Posters

Section:

3. Number Theory and Arithmetic Algebraic Geometry

1991 MS Classification: 11M06, 11M26, 11J99
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Reorderings of some divergent series and applications

Using some results of Skölem and Bang on Diophantine approximations it is shown that if $f : \mathbb{N} \to \mathbb{C}$ is a function such that $\sum_{n=1}^{\infty} |f(n) - \frac{1}{n}| < \infty$ and $\alpha \in [0,1]$ is irrational then $\sum_{n=1}^{\infty} \left(f\left(\left[\frac{n}{\alpha}\right]\right) + f\left(\left[\frac{n}{1-\alpha}\right]\right) - f(n) \right) = -\alpha \ln \alpha - (1-\alpha)\ln(1-\alpha)$, where $[x]$ denotes the integer part of x. This result is used to study some properties of the function $J(\beta) = \int_0^1 \rho\left(\frac{1}{x}\right) \rho\left(\frac{\beta}{x}\right) dx$, $\beta \in [0,1]$, $\rho(x) = x - [x]$, that arises in Beurlings reformulation of the Riemann Hypothesis as a problem of Functional Analysis.