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Section:

3. Number Theory and Arithmetic Algebraic Geometry

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Some new techniques in the study of automorphic and Hecke L-functions

Our goal is the meromorphic continuation to $\mathbf{C}^2$ of

$$Z(s,w) = \sum_{\substack{n \in \mathbf{Z}[i], \\ n \equiv 1(1+i)^3}} L_\lambda(s,\chi_n) L_\lambda(s,\bar{\chi}_n) N(n)^{-w},$$

where $\chi_n(m) = \left(\frac{n}{m}\right)_4$, for $(m,(1+i)n) = 1$ and $m \equiv 1(1+i)^3$, is the character associated to the cyclic extension $\mathbf{Q}(i, n^{1/4})/\mathbf{Q}(i)$; λ means that the Euler factor at $1+i$ has been removed.

Our approach is based on a functional equation relating $Z(s,w)$ to

$$\widehat{Z}(s,w) = \zeta_{\mathbf{Q}(i),\lambda}(2s+2w-1)\zeta_{\mathbf{Q}(i),\lambda}(4s-1)^2 \times \sum_{\substack{n = \lambda^t n_0, \\ n_0 \equiv 1(\lambda^3)}} \sum_{m,m' \equiv 1(\lambda^3)} \frac{G(n,m)\overline{G(n,m')}}{N(n)^w N(mm')^s}.$$

Using the theory of metaplectic Eisenstein series we prove the following

Theorem. *$\widehat{Z}(s,w)$ has a meromorphic continuation to the tube domain in $\mathbf{C}^2$ associated to the convex region bounded by $w=1$, $w=2-s$, $w=2-2s$. It has double poles at $s=\frac{1}{4}$ and $s=\frac{3}{4}$. Furthermore, up to a finite scattering matrix, it satisfies the functional equation*

$$\widehat{Z}(s,w) = \widehat{Z}(1-s, 2s+w-1).$$

As a consequence, the meromorphic continuation of $Z(s,w)$ is proved using the convexity principle in tube domains.