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Abstracts of Short Communications and Posters

Section:

3. Number Theory and Arithmetic Algebraic Geometry

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On the factors $\Phi^{(j\delta/m)}(x)$ of the period polynomial for finite fields.

Denote by $\mathbf{F}_q$ the finite field of q elements, where $q = ef + 1$ is a power of a prime p. Set $\delta = \gcd((q-1)/(p-1), e)$. The period polynomial $\Phi(x)$ of degree e for the finite field $\mathbf{F}_q$ is known to factor over the rational field $\mathbf{Q}$ as a product of δ polynomials, $\Phi^{(w)}(x)$ $(1 \leq w \leq \delta)$, each of degree e/δ. For f fixed, I obtained formulas for the beginning coefficients of $\Phi^{(\delta)}(x)$ expressed in terms of a certain counting function $\beta_K(n)$ for positive integers n, where K is the decomposition field of p in the cyclotomic field of f-roots of unity. These formulas are computationally impractical in general, but I later found closed form expressions or generating functions for $\beta_K(n)$ when $K = \mathbf{Q}$, which lead to explicit expressions for the beginning coefficients of $\Phi^{(\delta)}(x)$. Subsequently, I found similar results for the middle factor $\Phi^{(\delta/2)}(x)$, when δ is even. Here I examine the factors $\Phi^{(w)}(x)$, where $w = j\delta/m$, $m|\delta$ and $\gcd(j,m) = 1$, to obtain analogous results, particularly in the case $K = \mathbf{Q}$.