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The Numbers that Can Be Represented as the Sum of Four Squares (One Repeating Itself Three Times), in Three Different Ways

All numbers of the form $4(n^2+3n+3)$ $(n = 1, 2, 3, ..)$ can be represented in three different ways, viz:

$$3.1^2 + (2n+3)^2 = 3(n+1)^2 + (n+3)^2 = 3(n+2)^2 + n^2$$

The fact that the first square repeats itself three times leads to its geometrical representation in three dimensions. It gives a set of circles, with centre at the point $(1, 1, 1)$, cut by the plane $x + y + z = 3$ on the sphere $x^2 + y^2 + z^2 = 28$, [which is $4(n^2 + 3n + 3)$, for $n = 1$]. It will also represent a family of pairs of circles lying in the planes parallel to the aforesaid plane, at intervals of $3^{1/2}$, with their centres lying on the line $x = y = z$, which will be cut on the successive spheres $x^2+y^2+z^2 = 4(n^2+3n+3)$. A sub-class of these numbers can also be represented, retaining the same characteristics, in six different ways.