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Complex Dimensions of Fractal Strings and Oscillatory Phenomena in Fractal Geometry and Arithmetic

We briefly report on a forthcoming research monograph joint with M. van Frankenhuysen and to be announced in *Contemporary Mathematics.* We develop a theory of *'complex dimensions'* of fractal strings (one-dimensional drums with fractal boundary). These 'complex (fractal) dimensions' describe the geometric and spectral oscillations of fractal strings by means of 'explicit formulas'. We put the theory of Dirichlet series in this geometric context: their poles acquire the geometric meaning of 'complex dimensions'. We give a geometric reformulation of the generalized Riemann Hypothesis in this context, thereby extending our earlier work (M. L. L. & H. Maier, *J. London Math. Soc.* **52**, 1995, 15-34). We also prove that zeta-functions from a large class of Dirichlet series do not have zeros forming a vertical arithmetic progression, thereby extending Putnam's theorem.
Our work suggests to define 'fractality' as the presence of nonreal complex dimensions with positive real part.