ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

3. Number Theory and Arithmetic Algebraic Geometry

1991 MS Classification: 11

Nicolas, Jean-Louis, Université Claude Bernard, Lyon, France

On the maximum order of torsion elements in $GL(n, \mathbb{Z})$

The maximum order $g(n)$ of an element of the symmetric group S_n was first studied by E. Landau in 1903 who proved

$$\log g(n) \sim \sqrt{n \log n}$$

as $n \to +\infty$. A torsion element in $GL(n, \mathbb{Z})$ is a matrix M with integral coefficients, n rows and n colomns, and a finite order e such that $M^e = I$, the unit matrix. Let us denote by $G(n)$ the maximum order of these torsion elements. It is easy to see that $g(n) \leq G(n)$. In [1], with G. Levitt, we have proved

$$\log G(n) \sim \sqrt{n \log n}$$

and some other properties of $G(n)$, for instance, the largest prime number dividing $G(n)$ is asymptotic to $\sqrt{n \log n}$, as $n \to +\infty$.

Here, I shall prove the following result:

Theorem. *As $n \to +\infty$, one has $G(n)/g(n) \to +\infty$, and more precisely,*

$$(\log n)^{1+o(1)} \leq \frac{G(n)}{g(n)} \leq \exp((\log \log n)^{2+o(1)}).$$

Under Riemann hypothesis, the upper bound can be replaced by $(\log n)^{3+o(1)}$.

References: [1] G. Levitt and J. -L. Nicolas, On the maximum order of torsion elements in $GL(n, \mathbb{Z})$ and $\mathrm{Aut}(F_n)$, to appear, J. of Algebra.