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3. Number Theory and Arithmetic Algebraic Geometry

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Okada, Tatsuya, Fukushima Medical College, Sekiguchi, Takeshi, Tôhoku Gakuin University, Shiota, Yasunobu, Tôhoku Gakuin University, and Muramoto, Katsushi, Kawaijuku Educational Institution, Japan

Explicit formulas for a Generalized digital sum problem.

Let p be a positive integer greater than 1 and denote the p-adic expansion of $n \in \mathbf{N}$ by $n = \sum_{i\geq 0} \alpha_i(n)p^i$, where $\alpha_i(n) \in \{0,1,\ldots,p-1\}$. We set $s(n,l) = \sum_{i\geq 0} \mathbf{1}_{\{\alpha_i(n)=l\}}$ for $l = 1,2,\ldots,p-1$, and $\mathbf{s}(n) = (s(n,1),\ldots,s(n,p-1))$. We introduce the following generalization of the usual power and exponential sums. For $N \in \mathbf{N}$, $\boldsymbol{k} = (k_1,\ldots,k_{p-1}) \in \mathbf{N}^{p-1}$, and $\boldsymbol{\xi} = (\xi_1,\ldots,\xi_{p-1}) \in \mathbf{R}^{p-1}$, define $S_{\boldsymbol{k}}(N) = \sum_{n=0}^{N-1} \prod_{l=1}^{p-1} s(n,l)^{k_l}$ and $F(\boldsymbol{\xi},N) = \sum_{n=0}^{N-1} e^{\sum_{l=1}^{p-1}\xi_l s(n,l)}$. We shall give explicit formulas of $(\partial^{|\boldsymbol{k}|}/\partial\xi_1^{k_1}\cdots\partial\xi_{p-1}^{k_{p-1}})F(\boldsymbol{\xi},N)$ and $S_{\boldsymbol{k}}(N)$ for general p.
Furthermore, we apply these results to a problem of counting the number of occurrences of a fixed block of digits.