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Complex version of Catalan's problem

Catalan's problem consists of finding integer neighbors $j, j+1$ which are both powers of some - possibly different - integers ≥ 2. We will call such a pair a *Catalan pair*. The only pairs found so far are $(-1,0)$, $(0,1)$, $(8,9)$ and Eugène Catalań (Brügge, 1814 - Lüttich, 1894) conjectured 1844 that there are no other pairs. New research implies that if there is another pair at all, the corresponding powers must be very large. Details will be given in the lecture. The problem apparently lacks symmetry, in so far that a Catalan pair $(j, j+1)$ does not imply that $(-j-1, -j)$ is also a Catalan pair. If we look at $(8,9)$, then -9 is apparently not a square of a real number, but if we admit complex numbers, then $-9 = (3\,\mathrm{i})^2$ is a square of a complex integer. So by admitting complex integers, we obtain more symmetry. The definition of neighbors of complex integers will be introduced in a natural way. We not only gain symmetry, but we also find new pairs of complex integers showing that the real and complex problem of Catalan are not the same. There is also a small section on the density of the prime powers in the complex plane. We also find some complex integers, which are sandwiched between powers (Fermat numbers). (Coauthored by Wolfgang Ripken)