ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

4. Algebraic Geometry

1991 MS Classification: 14, 15, 30

Streit, Manfred

Field of Definition and Galois Orbits of some non–congruence Subgroups

Torsion–free normal subgroups of cocompact triangle groups Δ define Riemann surfaces X with large automorphism groups, though they are finite of order at most $84(g-1)$. This bound is attained for normal subgroups of Δ of signature $(2,3,7)$. X then is called Hurwitz curve. Macbeath shows: 1. $PSL_2(\mathbb{F}_q)$ is automorphism group of a Hurwitz curve for field characteristic p exactly for one p–power q. 2. The Hurwitz curve is uniquely determined by that group iff $p=q=7$ or $q=p^3$ and $p\equiv \pm 2$ or $p\equiv \pm 3 \bmod 7$. X then may be defined by equations with coefficients lying in $\mathbb{Q}$.

For $p=q\equiv \pm 1 \bmod 7$ we have 3 Hurwitz curves X_1, X_2, X_3. One can show: 1. Coefficients of a defining equaiton lie in $K := \mathbb{Q}(\zeta_7+\zeta_7^{-1})$, $\zeta_7 := \exp 2\pi i/7$ as the smallest possible field. 2. All equations are equivalent under a coefficientwise action of $Gal(\overline{\mathbb{Q}}/\mathbb{Q})$. 3. They contain no K–rational points for $p\equiv 1,5,9,13,17,25 \bmod 28$. A more general result for other non–congruence subgroups with a quotient equal to $PSL_2(\mathbb{F}_q)$ can be obtained.