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Section:

5. Differential Geometry and Global Analysis

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Nijenhuis tensors and obstructions for pseudoholomorphic mapping constructions

We present some approaches to classification of almost complex structures and to construction of local or formal pseudoholomorphic mapping from one almost complex manifold to another. The corresponding criteria are given in terms of Nijenhuis tensors and their generalizations.

We deal with the prolongations of the Cauchy-Riemann equation in 1-jets of the mapping from one almost complex manifold to another. We give a criterion of the prolongation existence of k-th pseudoholomorphic jets to the $(k+1)$-th ones. The main theorem on the solvability of the Cauchy-Riemann equation has the following form:

Let the mapping $u:(L,x)\to(M,y)$ between two almost complex manifolds be such that

$$j_M\circ u_*\equiv u_*\circ j_L(mod\mu^{k-1}),\ N_{j_M}\circ u_*^{\wedge 2}\equiv u_*\circ N_{j_L}(mod\mu^{k-1})\ (k\geq 2),\ (1)$$

where μ^k denotes the submodule of mappings $(L,x)\to(M,y)$ vanishing at $x\in M$ to the order k. Then there exists a mapping $\tilde{u}:(L,x)\to(M,y)$ such that

$$j_M\circ\tilde{u}_*-\tilde{u}_*\circ j_L\in\mu^k,\ u_*-\tilde{u}_*\in\mu^{k-1}.$$

As a consequence basing on the Kuranishi's approach we obtain formal normal forms of almost complex structures. As another consequence we get the formal part of Newlander-Nirenberg's theorem on integrability.

We also introduce the notion and study the properties of the linear Nijenhuis tensor, which we apply to almost complex structures of general position. As applications of this theory we exhibit two classifications: a classification of four-dimensional almost complex structures in terms of distributions and a characterization of the new class of structures which we call Lie almost complex structures.