ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

5. Differential Geometry and Global Analysis

1991 MS Classification: 58F07

Selivanova, Elena N., Universität Tübingen, Germany; Nizhny Novgorod State Pedagogical University, Russia

New integrable cases of conservative systems on S^2

We propose *new* examples of *conservative systems* on S^2 possessing an integral of *third* and *fourth* degree in momenta. As particular cases of our construction we obtain the known cases of Goryachev–Chaplygin (degree 3) and Kovalevskaya (degree 4). We prove

THEOREM 1. Conservative systems on S^2 with energy

$$H = \frac{d\varphi^2 + dy^2}{\psi'^2(y)} - \left(\psi''(y) - \psi(y)\right)\,\psi'^2(y)\cos\varphi$$

$$H_b = \frac{\psi'^2(y) - \psi^2(y) + b}{\psi'^2(y)}\left(d\varphi^2 + dy^2\right) - \frac{\psi'^2(y)\left(\psi''(y) - \psi(y)\right)}{\psi'^2(y) - \psi^2(y) + b}\cos\varphi\,,$$

where ψ is a solution of the initial value problem

$$x'x''' = xx'' - 2x''^2 + x'^2 + x^2\,, \quad x(0) = 0\,, \quad x'(0) = 1\,, \quad x''(0) = \tau\,, \quad \tau \in (0, T)$$

$$b > b_*(\tau) = \max_{y\in\mathbf{R}}(\psi^2 - \psi'^2) \text{ or } b < b^*(\tau) = \min_{y\in\mathbf{R}}(\psi^2 - \psi'^2)$$

and T is a certain positive constant, possess a nontrivial integral cubic in momenta.

THEOREM 2. Conservative systems on S^2 with energy

$$H = A^{-\frac{1}{2}}\left(d\varphi^2 + A^{-\frac{1}{2}}du^2\right) - \frac{u}{2}\left((a + 2u^2) - 2A^{\frac{1}{2}}\right)\cos\varphi\,, \quad -2 < a < 2\,, \quad a > 2\,,$$

$$H_p = \frac{A^{\frac{1}{2}} - u^2 + p}{A^{\frac{1}{2}}}\left(d\varphi^2 + A^{-\frac{1}{2}}du^2\right) - \frac{u\left((a + 2u^2) - 2A^{\frac{1}{2}}\right)}{2(A^{\frac{1}{2}} - u^2 + p)}\cos\varphi$$

where $A(u, a) = 1 + au^2 + u^4$ and either $a > 2$, $p \in (-\infty, -\frac{a}{2}) \cup (-1, +\infty)$ or $-2 < a < 2$, $p \in (-\infty, -1) \cup (-\frac{a}{2}, +\infty)$ possess a nontrivial integral of fourth degree in momenta.