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5. Differential Geometry and Global Analysis

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A Nonexistence Result of Harmonic Maps between Hadamard manifolds

Let $N = (N^n, g)$ be a *Hadamard manifold* i. e. a simply connected complete Riemannian manifold with non positive sectional curvatures. For some fixed $q_0 \in N$, let $\sigma(q_0, q)(t)$ be the geodesic curve such that $\sigma(q_0, q)(0) = q_0$ and $\sigma(q_0, q)(1) = q$. Let us define $k_{q_0}(q)$ and $\lambda_{q_0}(q)$ as : $k_{q_0}(q) := \max\{K(q\,;\pi) : \pi \ni \sigma'(q_0, q)(1)\}$, $\lambda_{q_0}(q) := \inf\left\{\frac{\|\xi\|^2_{g(q)}}{|\xi|^2}\ ;\ \xi \cdot \sigma'(q_0, q)(1) = 0\right\}$, where $K(q\,;\pi)$ denotes the sectional curvature of N at q with respect to a plane section π. The following result will be presented.

Theorem *Let $N = (N^n, g)$ be a Hadamard n-manifold which satisfies*

$$\{\mathrm{dist}(q_0, q)\}^2 |k_{q_0}(q)| \geq \kappa > 0 \quad as \quad \mathrm{dist}(q_0, q) \to \infty.$$

for some $q_0 \in N$. Then there exists no harmonic map $U : \mathbb{R}^m \to N$ which satisfies the following nondeneracy condition at infinity:

$$\liminf_{|x|\to\infty} \left(\frac{|x|^2}{\rho^2 \lambda_{q_0}(U(x))} \right) (e(U)(x) - e(\rho)(x)) > 0,$$

where $e(U)(x)$ denotes the energy density of U at x, $\rho(x) = \mathrm{dist}_N(U(x), q_0)$, and $e(\rho)(x) = \frac{1}{2}\sum_{\alpha=1}^m \left|\frac{\partial \rho}{\partial x^\alpha}\right|^2$.