# ICM 1998, Berlin, Aug. 18–27

## Abstracts of Short Communications and Posters

### Section:

## 7. Lie Groups and Lie Algebras

1991 MS Classification: 16, 17, 22
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**The exponential matrices of non-commuting matrices may commute**

We show how to construct entire families of non-commuting 2-by-2 real matrices $A$ and $B$ for which $e^A \cdot e^B \neq e^{A+B}$ and yet $e^A \cdot e^B = e^B \cdot e^A$ . Though it is known that $e^{sA} \cdot e^{tB} = e^{tB} \cdot e^{sA}$ , $\forall\ s,t \in \mathbb{R} \Longleftrightarrow A \cdot B = B \cdot A$ and that $e^{At} \cdot e^{Bt} = e^{(A+B)t}$ , $\forall\ t \in \mathbb{R} \Longleftrightarrow A \cdot B = B \cdot A$, our examples provide non-commuting 2-by-2 real matrices $A$ and $B$ for which $e^{At} \cdot e^B = e^B \cdot e^{At}$ , $\forall\ t \in \mathbb{R}$. We also give examples of non-commuting 4-by-4 real matrices (and, equivalently, of 2-by-2 complex matrices) $A$ and $B$ such that $e^A = e^B = e^{A+B} = I =$ the identity matrix and thus $e^A \cdot e^B = e^{A+B} = e^B \cdot e^A$.