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8. Analysis

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Inequalities of Landau–Kolmogorov–Nagy Type

We shall discuss connections of exact inequalities of Landau–Kolmogorov–Nagy type with various problems of Analysis, present new inequalities for univariate and multivariate functions and describe some of their applications. As an example we present here the inequality for having zeros univariate periodic functions x such that $x^{(r-1)}$ is locally absolutely continuous and $x^{(r)} \in L_\infty$:

$$\|x\|_q \le \sup_{0\le c\le \|\varphi_r\|_\infty} \frac{\|\varphi_r + c\|_q}{\|\varphi_r + c\|_p^{\frac{r+1/q}{r+1/p}}} \|x\|_p^{\frac{r+1/q}{r+1/p}} \left\|x^{(r)}\right\|_\infty^{\frac{1/p-1/q}{r+1/p}}, \ 0 < p < q \le \infty, \tag{1}$$

(Here, φ_r is the r^{th} periodic primitive with zero mean value over a period for $\varphi_0(t) = \operatorname{sgn}\sin t$.) The following result illustrates applications of (1):

$$\sup_{n\in\mathbf{N}} \sup_{\substack{T_n\in\mathcal{T}_n\\ T_n\ne 0}} \frac{\|T_n\|_q}{n^{1/p-1/q}\|T_n\|_p} = \sup_{c\in[0,1]} \frac{\|\cos(\cdot)+c\|_q}{\|\cos(\cdot)+c\|_p}, \ 0 < p < q \le \infty, \tag{2}$$

(where $\mathcal{T}_n$ is the set of all real-valued trigonometric polynomials of order n.) Relations (1) and (2) are joint work with V. A. Kofanov and S. A. Pichugov.