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Abstracts of Short Communications and Posters

Section:

8. Analysis

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Operator convex functions of two variables
To an $f : (-1,1)^2 \to \mathbb{R}$ there correspond *operator functions* $f : S_{(-1,1)}(\mathbb{C}^m) \times S_{(-1,1)}(\mathbb{C}^N) \to S(\mathbb{C}^M \otimes \mathbb{C}^N)$ where $S(X)$ denotes the set of self-adjoint operators on the inner product space $X, S_{(-1,1)}(X)$ those self-adjoint operators with spectrum in (- 1,1), and $M, N \in \mathbb{N}$. Then f is said to be *operator convex* if, for each $(M,N) \in \mathbb{N}^2, f((1-\theta)V + \theta W) \leq (1-\theta)f(V)+\theta f(W)$ for all $V, W \in S_{(-1,1)}(\mathbb{C}^M)\times S_{(-1,1)}(\mathbb{C}^n)$ and $\theta in[-1,1]$. The set $\mathcal{OC}_2$ of operator convex functions on $(-1,1)^2$ is a convex cone. A three parameter family of non-trivial finite dimensional faces of $\mathcal{OC}_2$ is identified. This contrasts with the one variable situation (v. F. Hansen and G. K. Pedersen, Jensen's inequality for operators and L"owner's Theorem, Math. Ann. 258: 229-241, 1982) in which the set of extreme points of a base of the convex cone of operator convex functions $g : (-1,1) \to \mathbb{R}$ form a topological interval.