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Section:

8. Analysis

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Relations between p-module and p-capacity in R^n

Let $\overline{R^n}$ be one-point compactification of Euclidean n-space R^n, E arbitrary sets, F closed sets, $\bar{E}$ the closure of E. The p-module of an arc family Γ is

$$M_p\Gamma = \inf_{\rho\in F(\Gamma)} \int \rho^p dm \quad (p>1),,$$

where $F(\Gamma) = \{\rho : R^n \to [0,\infty]$; Borel measurable and $\int_\gamma p dH^1 \geq 1\ \forall\gamma\in\Gamma\}$ and m and H^1 are n-dimensional Lebesgue and linear Hausdorff measures, respectively. $M\Gamma = M_n\Gamma$ is called the module of Γ. The p-capacity of $E_0, E_1 \subset \bar{D} \subset \overline{R^n}$ relatively to D is

$$\operatorname{cap}_p(E_0,E_1,D) = \inf_{u\in U} \int_D |\nabla u|^p dm \quad (p>1)\,,$$

where $U = \{u : D\cup E_0\cup E_1 \to [0,1]$; u continuous, $u_{|_{D-\{\infty\}}}$ locally Lipschitz, $u_{|_{E_0}} = 0$, $u_{|_{E_1}} = 1\}$ and ∇u is the gradient of u. $\operatorname{cap}(E_0,E_1,D) = \operatorname{cap}_n(E_0,E_1,D)$ is called the conformal capacity of the condenser (E_0,E_1,D). In this communication I establish some inequalities between the p-module and the p-capacity and also that

$$M_p\Gamma(F_0,F_1,D) = \operatorname{cap}_p(F_0,F_1,D) \quad (p>1)$$

if $F_0, F_1 \subset \bar{D} \subset \overline{R^n}$, $d(F_0,F_1) > 0$ and $\forall\xi\in\partial D\cap F_i$, for $i=0$ or $i=1$,

$$\lim_{x\to\xi, x\in D} \inf_\gamma H^1\left[\gamma(F_i,x)\right] = 0\,,$$

the infimum being taken over all arcs $\gamma = \gamma(F_i,x)$ joining F; and x in D $(i=0,1)$.