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Dilation equations and Hutchinson fractals

One way to solve the dilation equation
$\varphi(x) = \sum_{k=0}^{m} a_k \varphi(2x - k)$ is to construct from the *mask* $a_0, ..., a_m$ two $m \times m$ *dilation matrices*, P_0 and P_1, whose i, j-th elements are a_{2j-i-1} and a_{2j-1}, resp., where $a_k = 0$ for $k < 0$ or $k > m$. Under suitable conditions on the mask, the backwards products $P_{d_n} \ldots P_{d_1}$ converge for any sequence $d_1, d_2, ...$ of zeroes and ones to a continuous function f mapping $[0, 1]$ onto the flat Π_m in R^m made up of row-vectors whose components sum to one. The coordinates of f then determine φ.

We prove that from certain masks, including Daubechies' D_4, we can construct a translation-invariant metric on Π_m in such a way that each P is contractive with Lipschitz constant equal to its largest subdominant eigenvalue. This makes the graph of f into a *Hutchinson fractal* and has implications for the rate of convergence of the matrix products and for the Hoelderianity of φ.