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Abstracts of Short Communications and Posters

Section:

8. Analysis

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Haller, Rainis, Tartu University, Estonia

$M(r,s)$-inequality and reflexivity

Let $r \leq 1$ and s be positive numbers. We prove that a Banach space X satisfies the $M(r,s)$-inequality (i. e. $\|x^{***}\| \geq r\|\pi x^{***}\| + s\|x^{***} - \pi x^{***}\| \quad \forall x^{***} \in X^{***}$, where π is the canonical projection of X^{***} onto X^*) if and only if the unit sphere S_X of X has the following property: whenever $\epsilon > 0$, $x \in S_X$, and $(x_n)_{n=1}^{\infty} \subset S_X$, then there are $n_0 \in \mathbb{N}$, $u \in \operatorname{conv}\{x_1, \ldots, x_{n_0}\}$, and $v \in \operatorname{conv}\{x_{n_0+1},\ x_{n_0+2}, \ldots\}$ satisfying $\|rx + s(v-u)\| \leq 1 + \epsilon$. This immediately implies a well-known characterization of reflexivity due to R. C. James, generalizes criteria of M-ideals, and shows that the $M(r,s)$-inequality is separably determined. These results are joint work with E. Oja.

Reference: R. Haller and E. Oja, Geometric characterizations of positions of Banach spaces in their biduals, *Arch. Math.* **69** (1997), 227–233.