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8. Analysis

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The degree of rational approximation of a holomorphic function

Let $E = \{z : |z| \leq 1\}$, $F = \{z : |z| \geq \rho > 1\}$ and f be a holomorphic function on $D_\rho = \bar{\mathbf{C}} \setminus F$ with finite positive order σ:

$$\sigma = \limsup_{r \to \rho} \frac{\ln^+ \ln^+ \max_{|z|=r} |f(z)|}{-\ln(\rho - r)}.$$

Let $\rho_{n,m}$ be the distance of f in the uniform metric on E from the class of all rational functions of order at most (n, m). The results characterizing the rate of convergence to zero of the sequence $\{\rho_{n,m(n)}\}_{n=0}^{\infty}$, $m(n)/n \to \theta \in (0, 1]$ as $n \to \infty$ in terms of σ are obtained. Namely, the following upper estimates are valid:

$$\limsup_{n \to \infty} \frac{\ln^+ \ln^+ \left(\prod_{j=0}^{m(n)} \rho_{n-j,m(n)-j} \rho^{\theta n^2} \right)}{\ln n} \leq 1 + \frac{\sigma}{\sigma + 1},$$

$$\liminf_{n \to \infty} \frac{\ln^+ \ln^+ \left(\rho_{n,m(n)} \rho^{2n/(2-\theta)} \right)}{\ln n} \leq \frac{\sigma}{\sigma + 1}.$$