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8. Analysis

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Entire Functions of Finite Order and Regularly Varying Sequences

Regularly varying sequences (r. v. s) of index $\alpha \in R$ are defined as $c_n := n^{\alpha}L(n)$, where $L(n), n \in N$ are so-called slowly varying sequences in the sense of Karamata, satisfying: $\lim_n \frac{c_{[\lambda n]}}{c_n} = 1$ for each $\lambda \in R^{+}$. Examples of $L(n)$: $\ln^a 2n$, $\ln^b(\ln 3n)$, $e^{(\ln 3n)^c}$, $e^{\frac{\ln 2n}{\ln(\ln 3n)}}$, $\ldots$; $a, b \in R, 0 < c < 1$. We consider a set of entire functions $f(x) := \Sigma_{k=1}^{\infty} a_k x^k$ of finite order $\rho, 0 < \rho < +\infty$, with non-negative coefficients, and an operator $\widehat{f}$: $\widehat{f}(x) := \frac{xf'(x)}{f(x)}, x \in R^{+}$. In this paper we introduce a method for estimationg the asymptotic behavior of $f^*(x) := \Sigma_{k=1}^{\infty} c_k a_k x^k, x \to \infty$, and, among others, prove: (A:) If $\widehat{\widehat{f}(x)}$ is bounded from above, then $\frac{f^*(x)}{f(x)} \sim c_{[\widehat{f}(x)]}, x \to \infty$ for any r. v. s (c_k) of negative index. (B:) If $\widehat{f}(x) \in SR_{\rho}$ (smoothly varying functions of index ρ), then $\frac{f^*(x)}{f(x)} \sim \rho^{\alpha} c_{[\ln f(x)]}, x \to \infty$ for any r. v. s (c_k) of arbitrary index $\alpha \in R$.