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8. Analysis

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Matrix Moment Problem

Let $\sum = (\sigma(i,j))_{i,j=1}^N$ be a positive matrix of borel measures on the real line $\mathbb{R}$. We say that $\sum$ has all moments finite if

$$m_n(i,j) = \int_{\mathbb{R}} x^n \, d\sigma(i,j)(x)$$

are finite. In this case we define a sequence of matrix moments $M_n = (m_n(i,j))_{i,j=1}^N$. The matrix moment problem asks how to construct a matrix measure $\sum$ from a given sequence of matrix moments and if such a measure is unique. It is closely connected with the analysis of self-adjoint three-term difference operators acting on sequences of matrices. The uniqueness of solutions of the matrix moment problem is related to the dimensions of the kernel of $\Gamma(Z)$ as an operator on $\mathbb{C}^N$, where

$$\Gamma(z) = \lim_{n+\infty} \left(\sum_{k=0}^{n} P_k^*(z) P_k(z) \right)^{-1}$$

and $P_n(z)$ are the matrix polynomials orthogonal with respect to the following "quasi-scalar" product:

$$(F,G) = \int_{\mathbb{R}} F(x) d\sum(x) G^*(x) = \left(\sum_{k,l=1}^{N} \int_{\mathbb{R}} f(i,k)(x) \overline{G(j,l)(x)} d\sigma(k,l)(x) \right).$$

All matrix measures $\sum$ for which the matrix polynomials form a dense subset in $L^2(\mathbb{R}, \mathbb{C}^n; \sum)$ (they are so-called N-extremal measures) can be identified with unitary matrices acting on a subspace of $\mathbb{C}^N$.