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9. Ordinary Differential Equations and Dynamical Systems

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Integrability of functional equations defined by birational mappings

The algorithm is suggested for the transition from ODEs and PDEs to birational functional equations (BFEs) of the finite difference type in one and more complex variables defined by birational mappings of the group $\mathbf{BirCP}^n$ of all birational ones and the program of research associated with this approach is put forward. The first step of this program is as follows. The algebraic-geometry approach to integrability of BFEs is offered. The definition of BFEs is given as cascade-flows; and the theorem is formulated about the birational equivalence (BE) of birational plane mapping of degree n to the quadratic birational one. The definition of algebraic integrability of BFEs is given as the BE of initial BFEs to their linear part. We present the theorem on birational equivalence of an arbitrary birational mapping to its linear part that is a birational analog to the Poincaré and Siegel theorems on the holomorphic equivalence of a holomorphic mapping to its linear part. We obtain FEs for the mapping giving the birational equivalence and a rational invariant– first integral of initial BFEs. The Arnold complexity $C_A(k)$ of BFEs has the polynomially bounded growth with k for integrable plane mappings. An autonomous homogeneous linear difference equation is derived for C_A with integer coefficients. The conditions proving the polynomial boundedness of the growth of the Arnold complexity $C_A(k)$ define integrable surfaces in the space of all parameters of the initial birational mapping. The spectrum of eigenvalues of the associated characteristic equation defines integrability of initial BFEs. The nonalgebraic integrable BFEs are defined as the ones which are only holomorphically equivalent to their the linear part. There are different examples of algebraic integrable birational autonomous plane mappings (BFES), which are resonant and nonresonant, with invariants of genus one and zero, both constant in the phase space and constant only in the extended phase one. Nonalgebraic integrable BFEs, both resonant and nonresonant, were considered in previous papers.