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9. Ordinary Differential Equations and Dynamical Systems

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Integrability of functional equations defined by birational mappings

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The transition algorithm from ODEs and PDEs to birational functional equations (BFEs) of finite difference type in one and more complex variables defined by birational mappings of the group $\mathbf{BirCP}^n$ of all birational ones and the program of researches associated with this approach are suggested . Below, first step of this program is outlined. The algebraic-geometry approach to integrability of BFEs is proposed. The definition of BFEs as cascade-flows is given; the theorem about birational plane mapping of degree n a birationally equivalent (BE) to quadratic birational map is formulated. The definition of algebraic integrability of BFEs as BE initial BFEs to their linear part is proposed . We present a birational analog of the Poincare theorem and the Siegel one for the birational mapping birationally equivalent to its linear part. We obtain FEs for the mapping giving the birational equivalence and a rational invariant-first integral of initial BFEs. The Arnold complexity $C_A(k)$ of BFEs has the polynomially bounded growth with k for integrable plane mappings. An autonomous homogeneous linear difference equation for C_A with integer coefficients is derived. The conditions providing the polynomial boundedness of the growth of the Arnold complexity $C_A(k)$ define integrable surfaces in the space of all parameters of initial birational mapping. The spector of eigenvalues of the associated characteristic equation defines integrability of initial BFEs. For their integrability it is enough in order that the spector consist only from ± 1 values. The definition of nonalgebraic integrable BFEs as ones which are only holomorphically equivalent to their the linear part are given. There are different examples of algebraic integrable birational autonomous plane mappings (BFES) with invariants of genus one and zero, both constant in the phase space and constant only in the extended phase one, both nonresonant and resonant. Nonalgebraic integrable BFEs , both resonant and nonresonant, were integrated earlier in my papers Phys. D (1992), Phys. D (1995), J. of Math. Phys. (1998) using the Birkgoff theorem about normal form and the Moser

analytical invariants theorem of area-preserving (Poincare- resonant) plane mappings and the Baker-Feldman theorem about linear forms of logarithms of algebraic numbers.