ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

9. Ordinary Differential Equations and Dynamical Systems

1991 MS Classification: 34C25, 58E05,70F10, 70H05.
Hasna Riahi

Periodic Orbits of N-Body Type Problems via Morse Theory

We study the existence and multiplicity of periodic solutions of Hamiltonian systems of the form

$$(HS) \quad m_i\ddot{q}_i + \frac{\partial V}{\partial q_i}(t,q) = 0$$

where $q_i \in \mathbf{R}^\ell$, $\ell \geq 3$, $1 \leq i \leq n$, $q = (q_1, ..., q_n)$ and $V = \sum_{1\leq i\neq j}^{n} V_{ij}(t, q_i q_j)$ with $V_{ij}(t,\xi)$ T periodic in t and singular in ξ at $\xi = 0$. Under additional hypotheses on V, when (HS) is posed as a variational problem, we prove that the corresponding functional, I, has an unbounded sequence of critical values provided that the singularity of V at 0 is strong enough. The critical points of I are classical T-periodic solutions of (HS). Then, assuming that I has only non-degenerate critical points, up to translations, Morse type inequalities are proved and used to show that the number of critical points with a fixed Morse index k grows exponentially with k. When, the potential V is autonomous, we also study the existence and multiplicity of periodic orbits with fixed energy.

If we do not assume the strong force hypothesis at the singularity, we can only show the existence of generalized solutions to these problems, and then we study the number of collisions that such solutions may have.