ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

10. Partial Differential Equations

1991 MS Classification: 35

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New class of a priori estimates for nonlinear elliptic equations and systems of high order

We consider nonlinear elliptic systems of the form $div^m A(x, D^m u) = f(x)$, $x \in R^n$, with the natural energetic space W_2^m and standard structure conditions. For any $a \in (0, a^*)$ where $a^* \in (0, n)$ defined explicitely by the modulus of ellipticity of system we establish the estimate

$$\int \frac{|D^m u(x)|^2}{\omega(x)} \leq c \int \frac{|I_m f(x)|^2}{\omega(x)}, \qquad \omega(x) = 1 + I_{n-a}\sigma(x),$$

with arbitrary nonnegative finite Borel measure σ on R^n, constant c does not depend on σ; I_t is the fractional integral of order t. Applications to solvability in spaces that are weaker than the natural energetic space of system will be presented. In particular, for any $p > n/(n+m)$ the explicit restriction on the modulus of ellipticity provides the solvability of system for $f \in L_p$, $p \geq 1$, and for $f \in H_p$ (Hardy spaces), $p \leq 1$, is given (known results provide solvability only for $p = 2n/(n+2m)$).