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10. Partial Differential Equations

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Interior Hölder continuity of weak solutions to nonlinear parabolic systems in two spatial dimensions

Let $\Omega \subset \mathbb{R}^2$ be open, let $0 < T < +\infty$ and set $Q = \Omega \times (0,T)$. We consider the following system of nonlinear PDE's in Q:

$$\frac{\partial u^i}{\partial t} - \frac{\partial}{\partial x_\alpha} a_i^\alpha(x,t,u,\nabla u) = f_i(x,t) \qquad (i = 1,\dots,N) \tag{1}$$

where $u = \{u^1,\dots,u^N\}$, $\nabla u = \left\{\dfrac{\partial u^i}{\partial x_\alpha}\right\}$ (= matrix of spatial derivatives; $\alpha = 1,2$). The coefficients $a_i^\alpha = a_i^\alpha(x,t,u,\xi)$ are assumed to satisfy appropriate Hölder continuity conditions with respect to $\{t,u,\xi\}$. Let $f \in L^2(Q;\mathbb{R}^N)$ satisfy the condition

$$\int_{t_0}^{t_1}\int_{\Omega'} |f(x,t+h) - f(x,t)|^2 \mathrm{d}x\mathrm{d}t \le Ch^{2\mu} \quad \forall\, 0 < h < T - t_1,$$

$\forall\, 0 < t_0 < t_1 < T$, $\forall\, \Omega' \subset \overline{\Omega'} \subset \Omega$, where $\frac{1}{2} < \mu < 1$ is "near to 1".

Then we prove: *any weak solution* $u \in W_2^{1,0}(Q;\mathbb{R}^N)$ *with* $\operatorname*{esssup}_{t\in(0,T)} \int_\Omega |u(x,t)|^2 \mathrm{d}x < +\infty$ *is Hölder continuous in* Q.