ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

12. Probability and Statistics

1991 MS Classification: 60
Mattner, Lutz, Universität Hamburg, Germany

What are cumulants?

We characterize cumulants [= expectation, variance, ...] by their useful properties:

Let $n \in \{0, 1, \ldots\}$ and let $\mathcal{P}_n$ be the set of all probability measures P on $\mathbb{R}$ with $\int |x|^n \, dP(x) < \infty$. Consider $\mathcal{P}_n$ as a semigroup with respect to convolution. Topologize $\mathcal{P}_n$ by the weighted total variation distance $(P, Q) \mapsto \int (1 + |x|)^n \, d|P - Q|(x)$.

Theorem *The continuous homomorphisms from $\mathcal{P}_n$ into the additive group $\mathbb{R}$ are precisely the linear combinations of cumulants of order $\leq n$.*

Corollary *The scale equivariant ones are the constant multiples of cumulants.*

For $n = 0$, the Theorem yields a result of Halász [given in Rusza & Székely (1988), *Algebraic Probability Theory*, Wiley], namely:

The only continuous homomorphism on $\mathcal{P}_0$ is 0.