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0-1 Laws for Nonhomogeneous Markov Chains

We present a very simple but useful generalization of the classical Kolmogorov's 0-1 Law for a sequence of independent random variables. Let (Z_n) be a nonhomogeneous Markov chain with values in countable state space M, Φ be the corresponding "tail" σ-algebra, i. e. $\Phi = \cap_n F_{n\infty}$, where $F_{n\infty} = \sigma(Z_n, Z_{n+1}, ...)$ and event $A \in \Phi$. Obviously both $P(A)$ and $P(A|(Z_n = i)$ may take in general any values between 0 and 1. But for large n the trajectories of (Z_n) are in states where $P(A|(Z_n = i)$ is near 0 or near 1. The precise formulation is as follows. Denote by $M_n(p,q) = \{i \in M : p \leq P(A|(Z_n = i) \leq q\}$, $n \in N$, $0 \leq p \leq q \leq 1$.

Theorem 1. *For any $A \in \Phi$ and any $0 < p < q < 1$*
a) $\lim_n P(Z_n \in M_n(q,1)) = \lim_n P((Z_n \in M_n(q,1)) \cap A) = P(A)$,
b) $\lim_n P(Z_n \in M_n(0,p)) = 1 - P(A)$,
c) $\lim_n P(Z_n \in M_n(p,q) = 0$.

Theorem 1 was used in the proof of the Decomposition-Separation theorem describing the asymptotic behaviour of general finite nonhomogeneous Markov chain. [I. Sonin, IMS, Lect. Notes-Monograph Series, v. **30**, (1996), Stat., Prob. and Game Th., papers in Honor of David Blackwell, eds. T. S. Ferguson, L. S. Shapley and J. B. MacQueen, 337-346].