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13. Combinatorics

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Discrete convexity and unimodularity

A class $\mathcal{K}$ of discrete convex sets is defined as a collection of subsets of an integral lattice $M = Z^N$ with the following properties: 1) $\forall\ A \in \mathcal{K}$, $A = \mathrm{co}(A) \cap M$; 2) if $A \cap B = \emptyset$, A, $B \in \mathcal{K}$, then there exists an integral linear functional which separates A and B; 3) $\forall\ A \in \mathcal{K}$ any face of A belongs to $\mathcal{K}$; 4) $\forall\ m \in M$ and $\forall\ A \in \mathcal{K}$, $m + A \in \mathcal{K}$; 5) $A + B \in \mathcal{K}$ with any A, $B \in \mathcal{K}$. We prove that a class $\mathcal{K}$ satisfies 1)–5) iff there is a unimodular system of vectors $\mathcal{R}$ such that elements of $\mathcal{K}$ are sets of integral points of integral polyhedra of $\mathcal{P}(\mathcal{R})$. A polyhedron $P \in \mathcal{P}(\mathcal{R})$ iff the affine span of each face of P is parallel to a linear span of some subset of $\mathcal{R}$. For example, a class of integral polyhedra with a unimodular system $\mathbf{A}_N = \{\pm e_i, i \in N, e_i - e_j,\ i, j \in N\}$ coincides with well-known class of integral g-polymatroids.