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Optimization of projection methods for solving linear ill-posed problems

We investigate the efficiency of projection methods for solving ill-posed first kind Fredholm equations

$$Ax(t) := \int_0^1 a(t,\tau)x(\tau)d\tau = f(t) \qquad (1)$$

with normal solution from $Range(|A|^\nu)$, $0 < \nu \leq 1$. Suppose that instead of (1) we have a perturbed equation $A_h x = f_\delta$, where $A, A_h \in \mathcal{L}(L_2, L_2)$, $f, f_\delta \in L_2$, $\|A - A_h\| \leq h$, $\|f - f_\delta\| \leq \delta$. By $card(\nu, \delta, h, \mathcal{H}^r)$ we mean the minimal amount of discrete information in the form of Galerkin functionals guaranteeng optimal order of accuracy $O(\delta^{\nu/(\nu+1)})$ on the class of equations (1) with operators $A \in \mathcal{H}^r$ such that $\sum_{i,j=0}^r \int_0^1 \int_0^1 \left|\frac{\partial^{i+j} a(t,\tau)}{\partial t^i \partial \tau^j}\right|^2 dt d\tau < \infty$. We obtain that for sufficiently small δ and $h = O(\delta)$ it holds

$$c_1 \delta^{-1/r} \leq card(\nu, \delta, h, \mathcal{H}^r) \leq c_2 \delta^{-1/r} \log \frac{1}{\delta}.$$

The indicated optimal order of $card(\nu, \delta, h, \mathcal{H}^r)$ is attained by a projection method connected with some hyperbolic cross.