ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

17. Control Theory and Optimization

1991 MS Classification: 49, 35

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Riccati-type PDE and Multidimensional Calculus of Variations

Consider the Dirichlet-type functional with the integrand

$$J = \int_D (A_{ij}^{\alpha\beta}(t)\partial x^i/\partial t^\alpha \partial x^j/\partial t^\beta + 2C_{ij}^{\alpha}(t)x^i \partial x^j/\partial t^\alpha + B_{ij}(t)x^i x^j)dt.$$

Here D is the r–dimensional manifold with the boundary ∂D, $t \in \mathbb{R}^r$, $x \in \mathbb{R}^n$; $x(t)$ is a section of the vector bundle E with the base D. Consider also the following system of quadratic PDE

$$\partial/\partial t^\alpha (A_{ik}^{\alpha\beta} W_{j\beta}^k + C_{ji}^{\beta}) = B_{ij} - A_{ks}^{\alpha\beta} W_{i\alpha}^k W_{j\beta}^s, \quad i,j = 1,\dots,n.$$

Solutions to the equation define a connection W on the vector bundle E. A field of extremals of the functional J in E exists iff the curvature of the connection W equals zero:

$$\partial W_\alpha/\partial t^\beta \partial W_\beta/\partial t^\alpha = [W_\alpha, W_\beta].$$