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Abstracts of Plenary and Invited Lectures

Section:

0. Plenary Lectures

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Local index theory and higher analytic torsion

Our talk will explain refinements of the Atiyah-Singer index theorem and of the Riemann-Roch-Grothendieck theorem, which have been obtained using local index theoretic techniques. These methods permit the construction of secondary invariants, the analytic torsion forms, or analytic torsion currents, whose functorial properties will be described.
Here is a short example. Let $\pi : X \to S$ be a holomorphic submersion of complex manifolds with compact fibres Z. Let E be a holomorphic vector bundle on X. Let $R\pi_*(E)$ be the higher direct image of E. By Riemann-Roch-Grothendieck,

$$\mathrm{ch}(R\pi_*(E)) = \pi_*(\mathrm{Td}(TX/S)\mathrm{ch}(E)) \text{ in } CH(S). \tag{1}$$

Recall that a metric g^E on a holomorphic vector bundle E determines a holomorphic hermitian connection ∇^E. Let ω^X be a Kähler metric on TX. It induces a metric g^{TZ} on $TZ = TX/S$. Let g^E be a metric on E. Assume that $R\pi_*(E)$ is locally free. Let $g^{R\pi_*(E)}$ be the L_2 metric on $R\pi_*(E)$ one obtains via fibrewise Hodge theory. Using the superconnection formalism of Quillen, one contructs higher analytic torsion forms $T(\omega^X, g^E)$, such that

$$\frac{\overline{\partial}\partial}{2i\pi} T(\omega^X, g^E) = \mathrm{ch}(R\pi_*(E), g^{R\pi_*(E)})\pi_*[\mathrm{Td}(TZ, g^{TZ})\mathrm{ch}(E, g^{R\pi_*(E)})]. \tag{2}$$

The analytic torsion forms are secondary objects attached to the submersion $\pi : X \to S$ and to the given metrics. Adiabatic limit techniques are used to establish the functorial properties verified by the above objects.
This leads in particular to a refinement of the theorem Riemann-Roch-Grothendieck, the Gillet-Soulé Riemann-Roch theorem in Arakelov geometry.
In the context of de Rham theory, one can derive a form of Riemann-Roch-Grothendieck for flat vector bundles, and construct still mysterious secondary invariants.