ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

1. Logic

1991 MS Classification: 03C45
Newelski, Ludomir, University of Wrocław and Polish Academy of Sciences, Poland
Meager forking and m-independence

Assume $\mathfrak{C}$ is a large saturated model of a countable complete first-order theory T and A, B are countable subsets of $\mathfrak{C}$. For any $a \in \mathfrak{C}$ we define the type $tp(a/A)$ of a over A as the set of all A-definable subsets of $\mathfrak{C}$ containing a. $S(A)$ denotes the topological space of all such types over A. Basic open sets are of the form $\{p \in S(A) : X \in p\}$, where X is an A-definable subset of $\mathfrak{C}$. $S(A)$ is homeomorphic to a closed subset of Cantor set.
We say that a is algebraic over A if a belongs to a finite A-definable subset of $\mathfrak{C}$. We define the algebraic closure $acl(A)$ of A as the set of all elements of $\mathfrak{C}$ algebraic over A. $acl(A)$ is countable. For $p \in S(B)$, $p(\mathfrak{C}) = \{a \in \mathfrak{C} : tp(a/B) = p\}$ and we define the trace of p over A as the set $Tr_A(p) = \{tp(a/acl(A)) : a \in p(\mathfrak{C})\}$, a closed subset of $S(acl(A))$. Similarly we define $Tr_A(X)$ for any subset X of $\mathfrak{C}$.
Assuming T is stable, Shelah defined on $\mathfrak{C}$ the notion of forking independence, generalizing those of linear independence in a vector space and algebraic independence in an algebraically closed field. He used this notion in his classification theory, dealing mainly with uncountable structures. The geometric aspects of forking independence were crucial for the work of Zilber, Hrushovski and others on the fine structure of models of stable theories.
One of the main open problems of model theory is Vaught's conjecture, saying that if T has $< 2^{\aleph_0}$ (that is, *few*) countable models, then T has countably many of them. Shelah proved this conjecture for ω-stable theories (1984), and Buechler for superstable theories of finite rank (1993).
For a proof of Vaught's conjecture it is important to investigate the ways, in which a type $p \in S(A)$ may be non-isolated. This is closely related to the size of $Tr_A(p)$. When T is superstable, then it may happen that for some finite sets $A, B \subset \mathfrak{C}$, for some non-isolated type $p \in S(B)$, the set $Tr_A(p)$ is infinite. In order to approach Vaught's conjecture for arbitrary superstable theories it seems necessary to investigate such traces of types. In this lecture I will speak of this research.
Namely, I introduced meager forking, m-independence and other notions, designed to measure traces of types. Using these notions I proved that if T is superstable with few countable models, then traces of types can not be arbitrary closed subsets of the space of types, but rather are subject to some strong restrictions. For instance, they are traces of some definable

subsets of $\mathfrak{C}$. Developing the theory of m-independence enabled me to prove Vaught's conjecture for some superstable theories of infinite rank.

These notions may be useful not only for Vaught's conjecture, but also for description of arbitrary models of stable theories. I proved that in a small stable theory m-independence is the strongest notion of independence on finite tuples of elements of $\mathfrak{C}$, which has some natural properties (like e. g. symmetry, transitivity) and strengthens forking independence. Then I developed the theory of m-independence parallel to the theory of forking independence.

In particular, the notions of meager forking, meager types, $\mathcal{M}$-rank, $*$-finite and $*$-algebraic tuples, m-stable and m-normal theories correspond to forking, regular types, U-rank, imaginary elements, superstable and 1-based theories in the theory of forking independence.

I proved that if T is superstable with few countable models, then T is m-stable, m-normal and the $\mathcal{M}$-rank of any (trace of a) complete type is finite and not larger than the U-rank of this type.

We say that a type $p \in S(A)$ is regular if forking dependence is a combinatorial pregeometry on $p(\mathfrak{C})$. Moreover p is called locally modular if this pregeometry is locally modular. The existence of locally modular types is fundamental for geometric model theory. It turns out that in a small superstable theory the phenomenon of local modularity of some regular types is closely related to the topological structure of the space of types, via the notions of meager forking and meager type.

A surprising consequence of m-normality is that for any group G interpretable in a superstable theory T with few countable models, the quotient group G/G^0 is abelian-by-finite (G^0 is the connected component of G).

Introducing $*$-algebraic tuples allowed me also to extend the definition of m-independence to an arbitrary small theory (where forking independence is usually not applicable). This may allow us to apply (suitably modified) methods of geometric model theory in a new context.